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Mechanical metastructure with embedded phononic crystal for flexural wave attenuation

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Abstract

Destructive interference-based metamaterials have shown excellent characteristics in elastic wave manipulation and vibration attenuation. Nevertheless, challenges persist in the application due to limited space and lightweight design, as current metastructures require additional beam structure. To simplify the design of metamaterials for flexural wave manipulation, this paper presents a new class of embedded phononic crystal for manipulating flexural wave propagation in both one and two-dimensional space by taking advantage of destructive interference, which can effectively suppress the mechanical vibration of a beam structure with a broad band gap. The flexural wave dispersion characteristic in a non-uniform beam structure is derived based on the Euler–Bernoulli beam theory, and an embedded phononic structure with the mechanism of destructive interference is presented to demonstrate its effectiveness in mitigating mechanical vibration. Subsequently, four typical units of embedded phononic structures are designed for attenuating flexural wave propagation in a beam structure. Finally, both numerical simulations, including one and two-dimensional phononic crystals, and physical experiments are implemented to evaluate the performance of the presented metastructure for flexural wave manipulation, which indicates that the proposed embedded phononic structures can effectively mitigate structural vibration in the low-frequency domain. To the best of our knowledge, it is the first attempt to design the metabeam with embedded phononic structures by taking advantage of destructive interference.

Keywords: embedded phononic crystal, flexural wave attenuation, metastructure, mechanical vibration, bandgap

1. Introduction

This study presents a novel embedded phononic crystal for attenuating flexural wave propagation by using the mechanism of destructive interference. Mechanical vibration, a natural phenomenon, which widely happens in the field of aerospace, navigation, and railway transportation [1–3]. Generally, vibration as an undesired source that affects the functionality of the product and leads to fatigue failure of the key component, and more severely, it brings safety and property problems [4–6]. Therefore, comprehensively studying mechanical vibration absorbers for flexural wave attenuation is of vital significance. To further contribute this research area, a pipeline for designing embedded phononic crystals with the mechanism of destructive interference is presented in this paper.

Phononic crystals and metamaterials [7–9], a class of artificial periodical structures that exhibit excellent performance on vibration attenuation [10, 11] and noise reduction [12, 13], have been significantly explored and developed. Generally, there are two mainstream mechanisms to design metastructure, either Bragg scattering or local resonance [14–19]. With

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the help of heuristic optimization and gradient-based methods, Xie *et al* [20]. introduced an interval Chebyshev surrogate model-based genetic algorithm for topology optimization of phononic crystals with maximum band gaps under interval uncertainties. Dong *et al* [21]. utilized a genetic algorithm to design a two-dimensional phononic crystal with a square lattice for two objectives, maximization of the relative widths of adjacent bands without constraint and a predefined average density constrained first bandgap maximization. Liu *et al* [22]. proposed a solid isotropic material with a penalization-based topology optimization method to design a one-dimensional metastructure for elastic wave attenuation. Van *et al* [23]. presented a level-set topology optimization framework with an enriched finite element method for designing phononic crystals with smooth but clear boundaries. The optimization-based pipeline is effective and efficient for mitigating mechanical vibration by reasonably distributing the materials, nevertheless, it is still troublesome to obtain an optimal structure that satisfies specific band gaps, and the manufacturability of the metastructure is also challenging in practical application.

Locally resonant structure [24–26], a promising metamaterial for elastic wave attenuation within a low-frequency range, has drawn much attention to devote to this area. This type of metamaterial can construct structure satisfying the given band gaps by changing stiffness of the spring and mass of the resonator [27–29]. Wang *et al* [30]. numerically and experimentally investigated a metamaterial sandwich plate with periodically embedded plate-type resonators for suppressing mechanical vibration, the results show good agreement between the experimental measurements and the numerical analysis for the bandgap. Based on the inertial amplification mechanism, Zhou *et al* [31]. developed a high-static-low-dynamic-stiffness (HSLDS) resonator for attenuating flexural wave propagation in a beam structure, which is able to create a much lower band gap than a pure HSLDS resonator. Wang *et al* [32]. presented a metamaterial rod with resonators by using the mechanism of negative stiffness for manipulating elastic waves within a very low-frequency region. Even though the local resonator-based elastic metamaterials are of fantastic performance on mitigating mechanical vibration under excitation with low frequencies, enlarging the band gap for attenuating flexural wave propagation in an elastic beam is still challenging.

Destructive interference [33–35], a type of interference in which two interfering waves have a displacement in opposite directions, emerges as a new direction to analytically design metastructure for flexural wave attenuation in a wide band gap, which is a class of simple but effective metamaterial for wave manipulation that has been theoretically and experimentally validated in practice [36]. Liu *et al* [37]. designed a one-dimensional metastructure to manipulate shear wave propagation, and numerically and experimentally demonstrated the attenuation performance on transverse waves. Similarly, Yoon *et al* [38] detailed the flexural wave propagation theory and presented a kind of two-dimensional triangular phononic structure for bending wave attenuation by taking advantage of destructive interference, both the numerical analysis and experimental result demonstrate the effectiveness of the designed phononic crystal for suppressing mechanical vibration. To further reduce the size of the metastructure, Liu *et al* [39]. presented a composite sandwich-based functionally gradient phononic crystal for manipulating flexural wave propagation in the beam structure based on Euler–Bernoulli beam theory, which can effectively suppress the mechanical vibration of the structure in low-frequency domain.

Overall, the destructive interference-based metamaterials have demonstrated effective performance on flexural wave attenuation with a broad band gap. However, until now, this type of metamaterial has required the introduction of an additional beam structure to generate the phenomenon of destructive interference. Furthermore, the thickness of the external phononic structure should be kept the same as the host structure. Consequently, applying these metamaterials in fields such as precision instruments and aerospace, where installation space is limited and lightweight requirements are high, proves challenging, thereby limiting the flexibility and robustness of the metamaterials. The inadequate investigations and challenges urge this paper to further study the destructive interference-based metastructure for flexural wave attenuation.

Rather than introducing external beam structures for flexural wave manipulation, the main idea of the present work is to attenuate flexural wave propagation with embedded phononic structures by taking advantage of destructive interference. First, we derive the flexural wave dispersion theory in a freeform-based beam structure and present an embedded phononic crystal by using destructive interference law. Subsequently, typical units of phononic structures are designed and arranged to mitigate mechanical vibration in a broad bandgap. Finally, numerical analysis and physical experiments are conducted to illustrate the effectiveness and accuracy of the proposed method for flexural wave attenuation in a low-frequency range. The main ***contributions*** of this paper can be summarized as follows:

(i) A novel embedded phononic structure for attenuating flexural wave propagation in the metamaterial beam is presented. To the best of our knowledge, it is a new attempt to design destructive interference-based embedded phononic crystals for mitigating flexural waves in beam structures.
(ii) The theory of flexural wave propagation in a free-form beam structure is analytically derived and wave dispersion characteristic is numerically validated by taking advantage of destructive interference.
(iii) Compared with external phononic crystals, the presented metastructure does not need additional beam structure to achieve destructive interference for flexural wave attenuation, which improves the flexibility and applicability of the metamaterials in engineering applications.

The rest of this paper is organized as follows: section 2 introduces the theory of flexural wave propagation in freeform shape-based beam structure and the basic principle of embedded phononic crystal design. Then the numerical analysis and physical experiment are conducted to demonstrate the effectiveness and advancement of the proposed embedded phononic crystal structure in section 3. Finally, the conclusion and future work are presented in section 4.

2. Methodology

In this section, we first present the flexural wave propagation theory in non-uniform shape-based beam structure, then destructive interference is detailed to analytically design embedded phononic crystal, and the bending wave attenuation performance is numerically validated with an illustrative example.

2.1. Flexural wave in non-uniform beam

The governing equation of the flexural wave propagation in a non-uniform beam structure shown in figure 1 can be formulated in equation (1) [40],

$$\frac{\partial^4 \Psi(x,t)}{\partial x^4} + \frac{M}{D}\frac{\partial^2 \Psi(x,t)}{\partial t^2} = 0 \tag{1}$$

where Ψ is the transverse displacement of a beam, M $= \int \rho A(h(x))\mathrm{d}x$ is the mass of the cross-section, and D $= \int EI(h(x))\mathrm{d}x$ is the bending stiffness of the section, where ρ, A, E and I are the density, area, Young's module and area moment, respectively. The speed of the flexural wave in the structure at an arbitrary position x is defined in equation (2), where ω is the angular frequency

$$c(x) = \left[\frac{EI(h(x))}{\rho A(h(x))}\right]^{\frac{1}{4}} \omega^{\frac{1}{2}}. \tag{2}$$

It can be found that the speed of the flexural wave relates to the area and area moment of the cross-section. Thus, the wavelength on the beam structure at an arbitrary position x can be formulated as equation (3) by using the relationship between wave speed *c(x)* and frequency *f*. Similarly, it can be observed that the wavelength can be manipulated by changing the area and area moment of the cross-section.

$$\lambda(x) = 2\pi\left[\frac{EI(h(x))}{\rho A(h(x))}\right]^{\frac{1}{4}} \omega^{-\frac{1}{2}}. \tag{3}$$

2.2. Metastructure design with destructive interference

Different from local resonance or Bragg theory-based phononic crystals for flexural wave attenuation, destructive interference utilizes the phase difference of waves to suppress wave propagation by mitigating the amplitude of the resultant wave, which has been successfully applied in noise canceling, and electromagnetic wave manipulation. Inspired by the mechanism of destructive interference, which offers us the possibility to manipulate the flexural wave propagation by creating the phase difference of elastic waves.

Figure 2 shows a beam structure with an embedded phononic crystal. The flexural wave propagates along the beam

Figure 1. A sketch of a non-uniform beam structure with variable cross section.

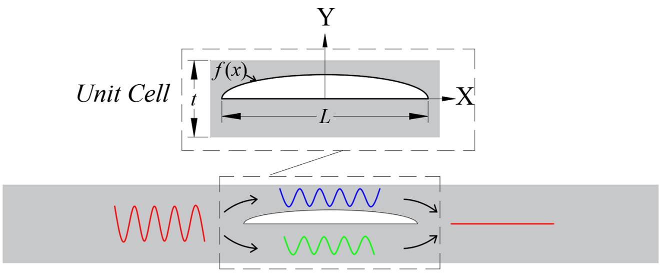


Figure 2. Schema of an embedded phononic crystal with destructive interference.

structure and divides into two waves at the separating point, then the flexural wave will propagate along two substructures and the destructive interference occurs when two waves come together in overlapping point with opposite direction. To realize the destructive interference for flexural wave manipulation in the beam structure, the substructures shown in the unit cell should satisfy the following physical rule [33, 41],

$$\delta = k_2 - k_1 = n + \frac{1}{2}, (n = 0, 1, 2, \ldots) \tag{4}$$

where $k_1 = L/\lambda_1$ and $k_2 = L/\lambda_2$ denote the wave number in parts 1 and 2, respectively, and λ_1, λ_2 indicate the average wavelength of the flexural wave in parts 1 and 2, respectively. The specific formulation of wavelength is shown as follows:

$$\lambda_i = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} g(x)\,\mathrm{d}x, (i = 1, 2) \tag{5}$$

where L denotes the length of the sub-beam structure, g(x) is the wavelength of the beam structure at position x, which is defined in equation (6), and t is the thickness of the beam, and *f(x)* is the function of the bottom boundary of the upper substructure. By adjusting the shape of the upper substructure, the unit cell can effectively mitigating the flexural wave with different target source frequency when follows the law in equation (4).

$$g(x) = 2\pi\left[\frac{E\left(\frac{t}{2} - f(x)\right)^2}{12\rho}\right]^{\frac{1}{4}} \omega^{-\frac{1}{2}}. \tag{6}$$

To illustratively demonstrate the wave attenuation performance of the metastructure, an embedded phononic structure based on quadratic function is given to validate the accuracy and effectiveness of the proposed method. The metastructure for evaluating the attenuation performance on flexural wave propagation is given in figure 3, and the quadratic function gives $f(x) = bx^2 - \frac{1}{4}bL^2$, where $b = 0.266$, and $L = 0.1$.

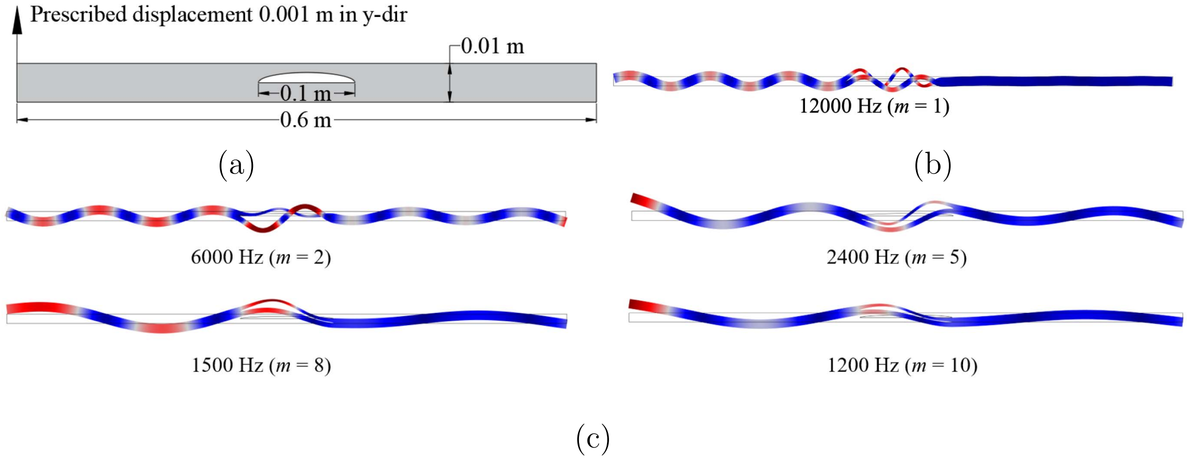


Figure 3. A destructive interference example for flexural wave attenuation in a beam structure. (a) the geometric shape and boundary condition, (b) wave propagation at 12 000 Hz source excitation, and (c) wave destructive with $\lambda^* = m\lambda$ ($m = 2$, 5, 8 and 10).

Figure 3(a) shows the geometric shape and boundary condition of the metabeam, specifically, the length and thickness of the beam structure are 0.6 m and 0.001 m, respectively, the Young's modulus E, density ρ, and Poisson ratio ν are 70 Gpa, 2700kg m^{-3}, and 0.33, respectively, and the left edge is applied with transverse vibration displacement. As illustrated in figure 3(b), the flexural wave is perfectly attenuated with the help of embedded phononic crystal under the excitation of a 12 000 Hz source excitation. Figure 3(c) shows the structure responses when the excitation of source frequencies satisfies the following relationship [39],

$$\lambda^* = m\lambda, \quad (m = 1, 2, 3, \ldots). \tag{7}$$

3. Numerical analysis and experiment

In this section, we first evaluate the effectiveness of the proposed embedded phononic crystal in a one-dimensional beam structure, and a two-dimensional phononic crystal is also designed to illustrate the performance on manipulating the flexural wave propagation. Then, an embedded phononic structure is designed and manufactured, and both the numerical simulation and physical experiment are conducted to demonstrate the effectiveness of the proposed metastructure for mitigating mechanical vibration. Here, COMSOL Multiphysics 6.0 is utilized for numerical simulation.

3.1. Numerical analysis

To demonstrate the effectiveness of the proposed method for suppressing mechanical vibration, a ellipse function shown in equation (8) is adopted, the four typical ellipse-type phononic structures are constructed to attenuate flexural wave propagation in the specific frequencies, i.e. 900 Hz, 1200 Hz, 1500 Hz, 1800 Hz, respectively, which are shown in figure 4(a), and the parameter details are summarized in figure 4(b)

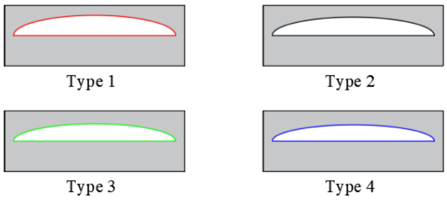
$$\frac{4x^2}{L^2} + \frac{y^2}{a^2} = 1. \tag{8}$$

3.1.1. One dimensional Embedded phononic crystal. To illustrate the effectiveness of the proposed method for attenuating flexural wave propagation, a one-dimensional beam structure with embedded phononic crystal is designed for performance comparison. Figure 5(a) presents beam structures and their boundary conditions, where the left edge is applied with transverse displacement in the Y axis, and the right side of the structure is selected to measure the response under excitations with various source frequencies. The structure deformation at a source excitation of 1350 Hz is shown in figure 5(b). It can be observed that the presented embedded phononic crystal can effectively attenuate the flexural wave propagation in the beam structure.

Figure 6 shows the attenuation performance on flexural wave propagation under various excitations, the wave attenuation curve is depicted by equation (9), where u_e and u_r denote the excitation displacement of the left edge and response displacement of the right edge, respectively [42–44]. It can be observed that the embedded phononic crystal can attenuate mechanical vibration in the designed bandgap when compared with the host structure, which demonstrates the effectiveness of the proposed method for manipulating flexural wave propagation in a one-dimensional beam structure

$$T = 20\log_{10}\left(\frac{u_r}{u_e}\right). \tag{9}$$

3.1.2. Two dimensional Embedded phononic crystal. To further demonstrate the effectiveness of the proposed method, a two-dimensional beam structure is constructed with designed phononic crystals. Figure 7(a) shows the geometric shape of the metabeam structure, of which a basic pattern is an octagon unit. The bottom edges are applied with transverse displacement along both the x and y axes, and the upper edges are used to measure the structure response,

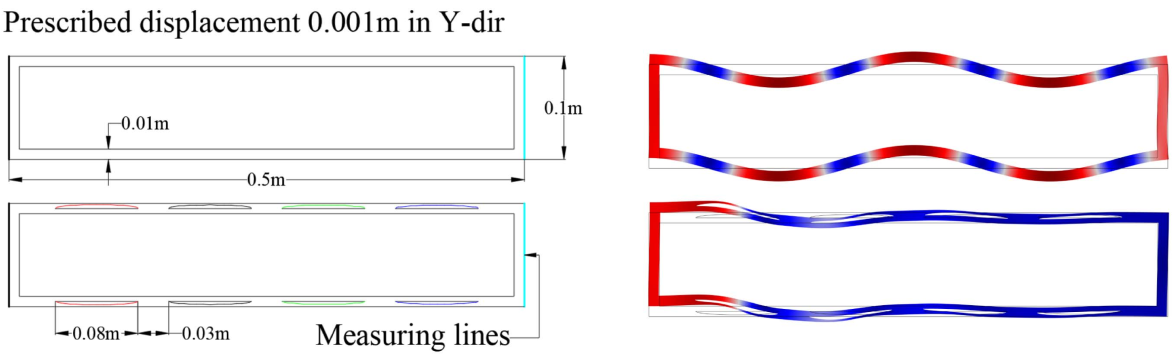


(a)

Type	L (m)	a (mm)	f_{target} (Hz)
Unit cell 1	0.08	3.332	900
Unit cell 2	0.08	3.064	1200
Unit cell 3	0.08	2.860	1500
Unit cell 4	0.08	2.697	1800

(b)

Figure 4. (a) Four different phononic structures for attenuating flexural wave propagation and (b) the specific parameters of each unit cell (m = 10).



(a) Geometric size (b) Structure deformation

Figure 5. One-dimensional metabeam structure.

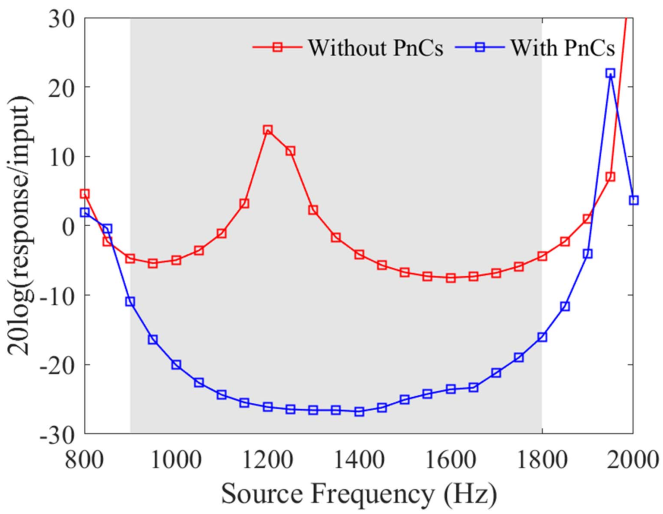


Figure 6. Structure response under various source frequencies.

and figure 7(b) gives the structure deformation under excitation with a 1350 Hz source frequency. It clearly shows that the host structure without embedded phononic crystals will vibrate heavier than the metastructure, namely, the proposed phononic pattern can effectively attenuate the flexural wave propagation in beam structure. Figure 8 records structure response under the excitations with various source frequencies, the energy transmission results indicate that the designed phononic crystal can manipulate flexural wave propagation in two-dimensional beam structure.

To further validate the effectiveness of the proposed phononic crystal for flexural wave attenuation in two-dimensional beam structures, another structure is designed to demonstrate the performance of proposed embedded phononic crystals on suppressing mechanical vibration. Figure 9(a) shows the geometry and boundary condition for numerical

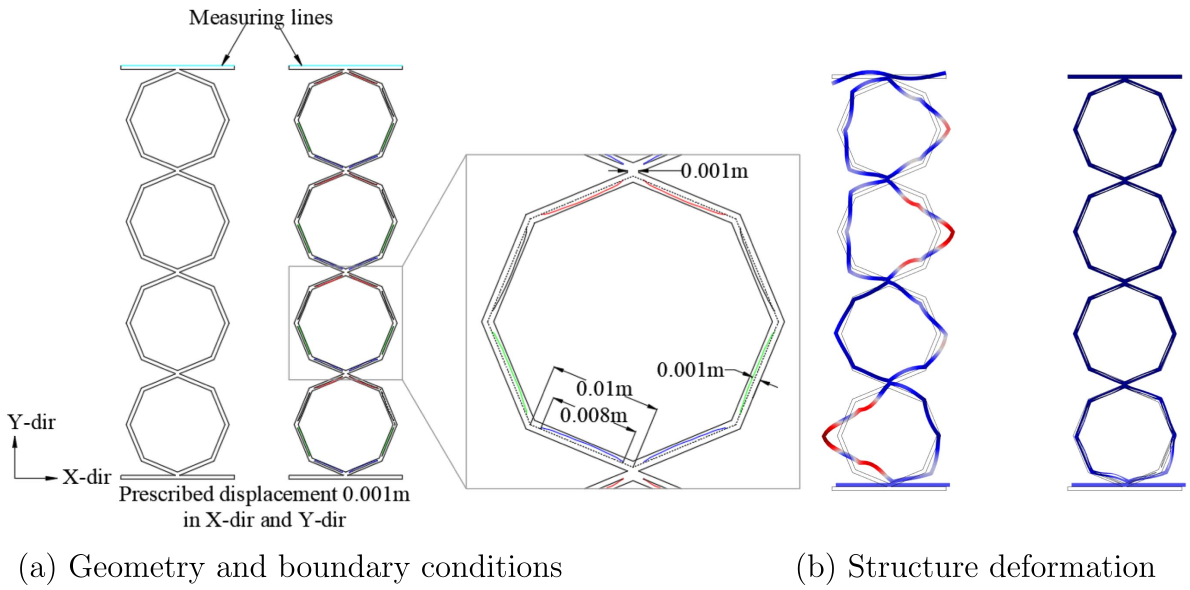


Figure 7. Two-dimensional metabeam structure.

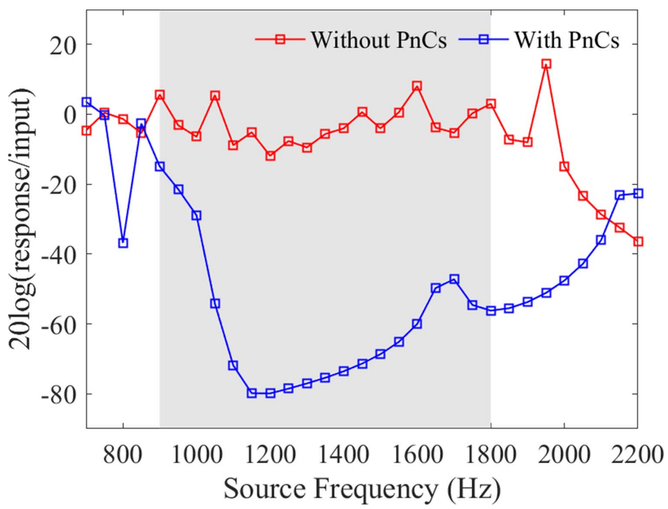


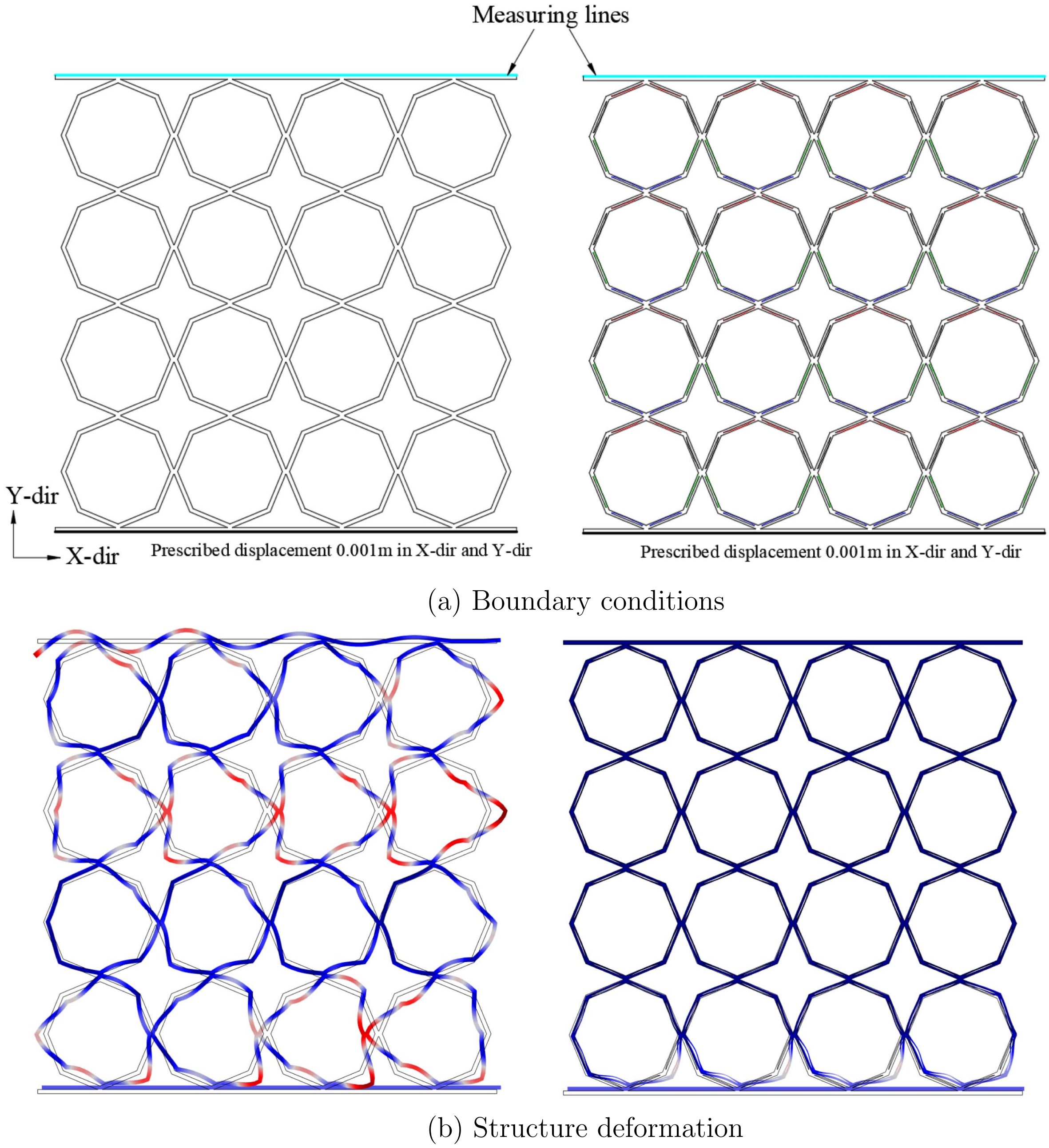
Figure 8. Structure response under various source frequencies.

simulation, and subplot (b) in figure 9 gives the structure deformation at a source excitation of 1350 Hz. Similarly, it can be observed that the host structure without phononic crystal can not attenuate the flexural wave propagation in the beam structure, while the metastructure can effectively attenuate the flexural wave propagation in the beam structure in both X and Y directions. The transmission curve shown in figure 10 shows that the embedded phononic crystal can effectively manipulate the flexural wave propagation in the desired bandgap.

Overall, the numerical simulations indicate the proposed embedded phononic structures are of good performance on manipulating the flexural wave propagation in both the one- and two-dimensional beam structures, and the mechanical vibration can be effectively suppressed in the desired frequency range when compared with the beam structure without embedded phononic crystals.

3.2. Physical experiment

To validate the physical performance of the designed phononic crystal on flexural wave attenuation, an aluminum-based prototype of the metastructure is manufactured by water-jet cutting technology, and a physical experiment is conducted to verify the effectiveness of the presented metastructure, of which the specific parameters of the embedded phononic crystal for the experiment is given in table 1. Subplot (a) and (b) in figure 11 shows the layout of the designed phononic structure and manufactured metastructure, respectively. Subplot (c) presents the experimental equipment including a desktop computer, impact hammer, acceleration sensors, and a DAQ module. In the experiment process, the hammer is used to apply point force at the upper left point of the structure, and the acceleration sensors are installed on the right edge to measure the response. Then, the measured data are transmitted to the



(a) Boundary conditions

(b) Structure deformation

Figure 9. Two-dimensional metastructure for wave manipulation.

desktop by DAQ module for signal analysis. Finally, Fourier transform is unitized to transform the time-domain signal to frequency-domain data.

Figure 12 shows the response of the manufactured metastructure under point load excitation. It can be found that the experimental result performs consistently with the numerical simulation at the designed frequency domain, namely, the metastructure successfully suppresses the propagation of the mechanical vibration, which demonstrates the effectiveness of the proposed method for flexural wave manipulation.

With the help of the numerical simulation and experiment, the proposed embedded phononic crystal shows the capacity to manipulate the flexural wave propagation in both one and two-dimensional beam structures. Overall, we can conclude that the proposed method can efficiently design embedded phononic crystal and the designed metastructure can effectively attenuate the flexural wave propagation in beam structure.

4. Conclusion and future work

An embedded phononic crystal is proposed to attenuate flexural wave propagation by taking advantage of destructive interference in this paper, which can effectively manipulate flexural wave propagation in a board band gap by arranging

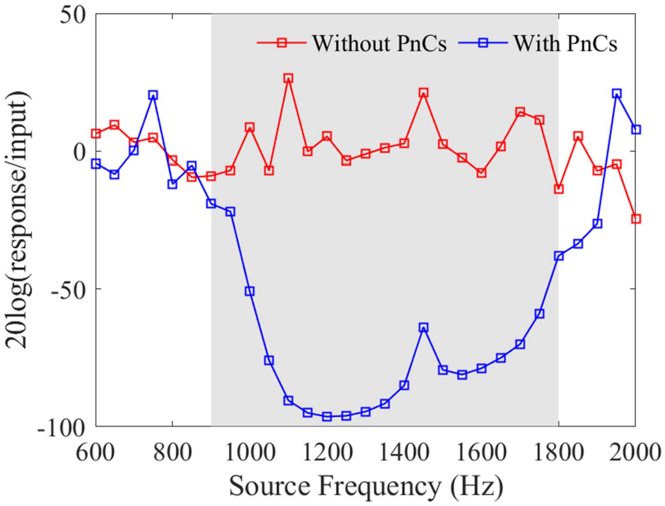


Figure 10. Structure response under various source frequencies.

Table 1. Specific parameters for the manufactured embedded phononic crystal.

Type	L (m)	t (m)	a (mm)	f_{target} (Hz)
Unit cell 1	0.12	0.02	5.622	900
Unit cell 2	0.12	0.02	5.115	1200
Unit cell 3	0.12	0.02	4.737	1500
Unit cell 4	0.12	0.02	4.440	1800

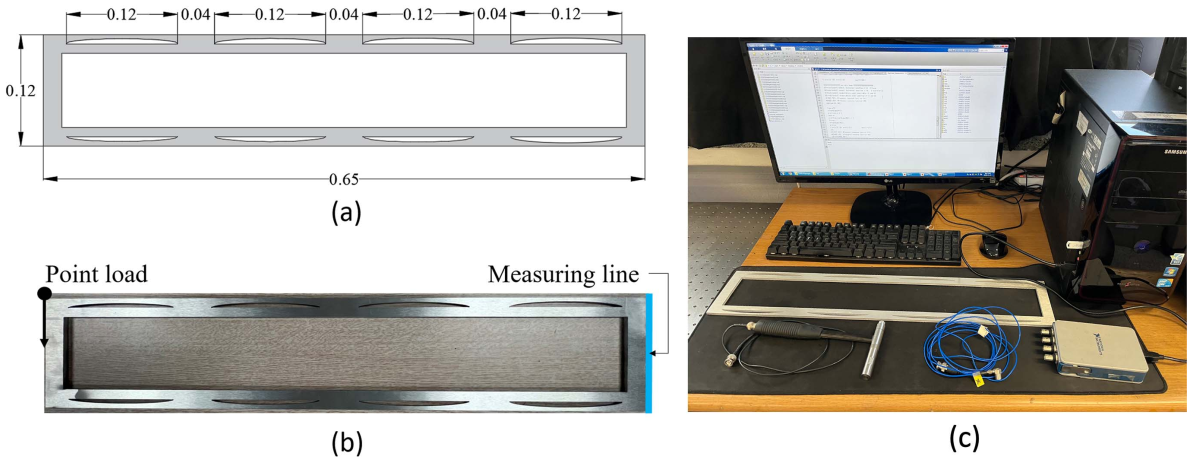


Figure 11. Physical experiment on an embedded phononic structure. (a) Geometric details of the phononic structure (unit: (m), (b) Physical prototype of the metastructure, (c) Experiment set-up.

embedding phononic patterns in the host structure. We first derive the flexural wave propagation theory in non-uniform beam structure, and the destructive interference is utilized to design embedded phononic patterns for manipulating flexural wave propagation. Then, four typical unit cells of phononic structure are given for mitigating beam structure vibration. Finally, both the numerical analysis and physical experiment are conducted to demonstrate the effectiveness of the proposed method for attenuating flexural wave propagation in beam structure. The proposed mechanical metamaterial for flexural wave manipulation can achieve destructive interference without addition beam structure when compared with external phononic crystals, which improves the applicability of the metastructure in practical application.

The proposed embedded phononic crystal for bending wave manipulation can be applied in the key components design of acrobat and mechanical products, and further work will consider structure stiffness and buckling performance in phononic crystal design, also, multi-materials-based metastructure design is critical to realize functionally gradient characteristic, which will be studied in future work.

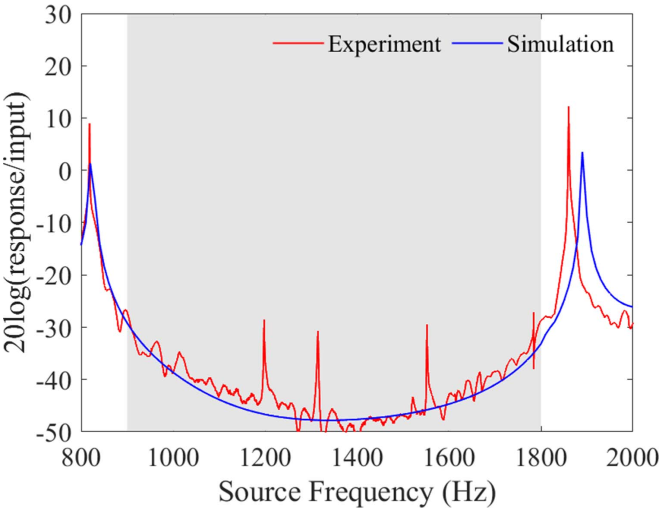


Figure 12. Response curve of the phononic structure.

Data availability statement

The data cannot be made publicly available upon publication because no suitable repository exists for hosting data in this field of study. The data that support the findings of this study are available upon reasonable request from the authors.

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Conflict of interest

The authors have no conflicts to disclose.

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