Numerical and Experimental Studies of Pendulum Dynamic Vibration Absorber for Structural Vibration

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This research presents a pendulum dynamic vibration absorber (PDVA) consisting of a spring and a mass in order to attenuate structural vibrations at two frequencies of hosting structure. It is a convention to attach several dynamic absorbers to hosting structure for the sake of the attenuations of structural vibrations at multiple frequencies with enlarged bandwidth and often it increases the total mass and the installation cost. Therefore, the reduction of the number of vibration absorbers for multiple excitation frequencies is an important issue from an engineering point of view. To resolve these difficulties, this study proposes to adopt the vibration absorber framework of the spring-mass vibration as well as the pendulum vibration simultaneously with the present PDVA system. It is composed of a spring and a mass but being allowed to swing circumferentially, the structural vibrations at the two resonance frequencies, i.e., the square root of stiffness over mass and the square root of a length over gravidity, can be simultaneously attenuated. As the length of the spring of the present PDVA is varied, the effective ranges for the pendulum dynamic vibration absorber become widen. To prove the concept of the present PDVA, this research conducts several numerical simulations and experiments. [DOI: 10.1115/1.4046951]

Keywords: pendulum system, spring-mass system, vibration absorber, multifrequency vibration attenuation, structural dynamics and control, vibration control, vibration isolation

1 Introduction

This research presents a new type of pendulum dynamic vibration absorber (PDVA) shown in Fig. 1 consisting of a spring and a mass to attenuate structural vibrations at two resonance frequencies of hosting structure. It is a convention to attach many dynamic absorbers to a structure to attenuate vibration at multiple frequencies and inevitably it increases the mass and the cost. Therefore, the reduction of the number of vibration absorbers at multiple excitation frequencies is an important issue. To reduce the number of vibration absorbers, this research employs the pendulum phenomenon with a dynamic vibration absorber consisting of a mass and a spring whose length is subject to be changed during vibration. As the present PDVA is composed of a spring, a mass, and damper, the structural vibrations at the two resonance frequencies, i.e., the square root of stiffness over mass and the square root of a length over gravidity, can be simultaneously attenuated with PDVA. As the length of the spring of the present PDVA is varied due to its vibration, it is also possible to increase the frequency ranges of the dynamic vibration absorbers. To show the efficiency of the present PDVA, this research carries out several numerical simulations and experiments.

Many relevant types of research for dynamic absorber exist [1–10]. Commonly tuned mass damper (TMD) or dynamic vibration absorber (DVA) has been widely used to attenuate structural vibrations. TMD refers to a dynamic absorber with mass, spring, and damper (see Refs. [1,2] and the references therein). In Refs. [11–14], the engineering applications of the pendulum dynamic vibration absorber can be found, i.e., building vibration, offshore structure vibration, wind vibration. In Ref. [15], a stepping motor and a microcontroller were used to control the resonance frequency and the damping ratio of DVA. In Ref. [16], the self-adjustable variable pendulum tuned mass damper was developed. In Ref. [17], the modal identification method identifying the mode shapes of a PDVA using an extended Kalman filter was studied. In Refs. [10,18], the PDVA systems for continuous structures were proposed [10,18]. In Refs. [19,20], the optimization scheme was applied to find out optimal parameters of PDVA considering the characteristics of the hosting structure. In Refs. [21–25], the nonlinearity of the pendulum tuned mass damper system was considered to attenuate the structural vibration. In Refs. [26,27], the dynamic response analysis of two PTMDs was suggested with the Van der Pol method. In Refs. [28,29], a torsional damper capable of suppressing torsional vibration was presented. In Ref. [30], the dynamic absorber utilizing pendulum and the

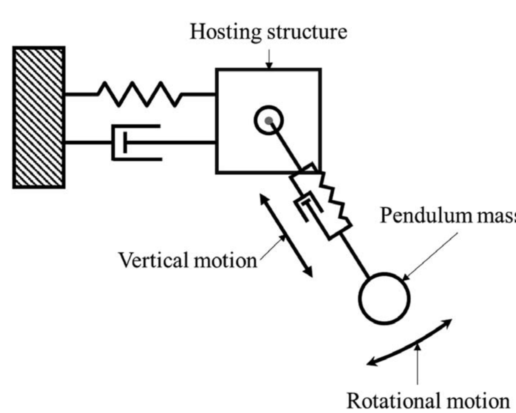

Fig. 1 Motion of pendulum dynamic vibration absorber (PDVA)

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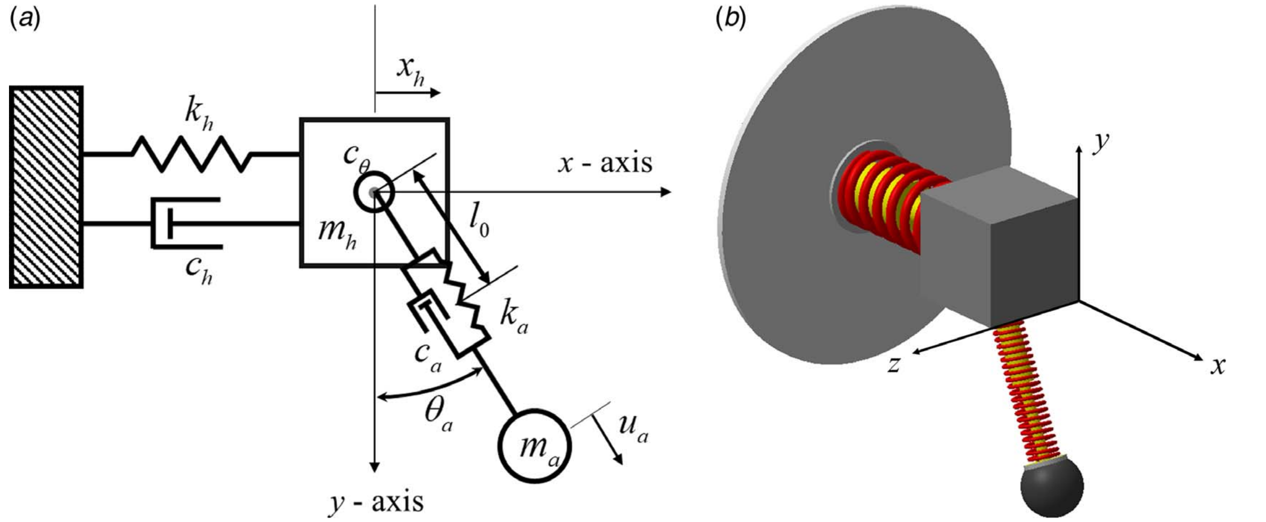

Fig. 2 The schematic diagram of the present pendulum dynamic vibration absorber (m_h and k_h: the mass and stiffness of hosting structure, m_a and k_a: the mass and stiffness of present pendulum dynamic absorber, θ_a: the angle between the hosting structure and the present dynamic absorber, x_h and u_a: the displacements of the hosting structure and the dynamic absorber, c_h, c_θ, and c_a: the damping coefficients, l_0: the initial length of the dynamic absorber)

mass-spring system have been proposed. The concept of the present study is similar to this research in Ref. [30]. One of the different points of the present PDVA compared with the dynamic absorber in Ref. [30] lies in the fact that the length of the spring is varied with respect to time. To contribute to these researches, this study presents a new type of dynamic absorber with a nonlinear pendulum (varying length) and a spring-mass system. Compared with the relevant researches, the present dynamic absorber consists with a nonlinear pendulum and spring-mass system. The spring contributes to not only a stiffness term but also a varying length string term of a pendulum as shown in Fig. 2.

In this research, we introduce a PDVA that is composed of a spring, a mass, and a damper, the structural vibrations at the two resonance frequencies, i.e., the square root of mass over stiffness and the square root of gravidity over length, can be simultaneously attenuated with one spring-mass system or PDVA. The present PDVA can attenuate structural vibrations at multiple frequencies of hosting structure. The length of the spring of the present PDVA is varied due to its vibration [31], and it is also possible to increase the frequency attenuation ranges without increasing the mass. To verify the efficiency of the present PDVA, this research carries out several numerical simulations and experiments.

The paper is organized as follows: Sec. 2 presents the new dynamic absorber with nonlinear pendulum and spring-mass system (PDVA). Section 3 conducts several numerical simulations and experiments to show the benefits of the present absorber. The conclusions and future topics are summarized in Sec. 4.

2 Pendulum Dynamic Vibration Absorber Using Pendulum and Mass-Spring System

To illustrate the concept of the pendulum dynamic vibration absorber using the resonances of pendulum as well as mass-spring system, let us consider the schematic of a vibration system with pendulum and mass-spring system in Fig. 2. Note that the mass of the pendulum is vibrating radially and vertically simultaneously; it is common to use only one of the motions but this research suggests to use the radial and vertical motions simultaneously for the accumulated attenuations. Although it is similar to the mass-spring vibration absorber, the present PDVA uses the vibrations in the two directions, i.e., vertical motion and radial motion. When the primary structure moves horizontally, the structure is excited vertically and horizontally. The excitation of the PDVA is dependent on the angle. For example, with 90 deg for θ_a, the radial excitation of the spring-mass system at the angular velocity, $\sqrt{k_a/m_a}$, is minimized whereas with 0 deg for θ_a, it is maximized. On the other hand, the motion of u_a is maximized with 0 deg for θ_a, and minimized with 90 deg for θ_a.

2.1 The Governing Equations for One Degree-of-Freedom Hosting Structure (Primary Structure).

For the governing equation of the system in Fig. 2, the location of the mass is expressed by considering the displacement, u_a, and the rotation, θ_a, as follows:

$$\text{Mass position of PDVA: } x_a = x_h + (l_0 + u_a)\sin\theta_a, \quad y_a = (l_0 + u_a)\cos\theta_a \tag{1}$$

where x_a and y_a represent the horizontal and vertical coordinates of the mass of the present PDVA, respectively. Without the loss of generality, the governing equation of the system can be obtained by applying the Euler–Lagrange's principle.

$$\text{Kinetic energy: } T = \frac{m_h\dot{x}_h^2}{2} + \frac{m_a(\dot{x}_a^2 + \dot{y}_a^2)}{2} \tag{2}$$

$$\text{Potential energy: } V = \frac{k_h x_h^2}{2} + \frac{k_a u_a^2}{2} + m_a g(l_0 + u_a - y_a) \tag{3}$$

$$\text{Dissipation energy: } F = \frac{c_h\dot{x}_h^2}{2} + \frac{c_\theta\dot{\theta}_a^2}{2} + \frac{c_p\dot{u}_p^2}{2} \tag{4}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_h}\right) - \frac{\partial L}{\partial x_h} + \frac{\partial F}{\partial \dot{x}_h} = 0,\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_a}\right) - \frac{\partial L}{\partial \theta_a} + \frac{\partial F}{\partial \dot{\theta}_a} = 0,\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{u}_a}\right) - \frac{\partial L}{\partial u_a} + \frac{\partial F}{\partial \dot{u}_a} = 0 \tag{5}$$

In Eq. (3), the potential energy for the absorber is $m_a g(l_0 + u_a - y_a)$ in order to consider the displacement of the absorber u_a. In addition, the force term is neglected as the initial velocity condition is imposed in the present study. From the Euler–Lagrange equations of Eq. (5), the three differential equations can be obtained.

$$(m_h + m_a)\ddot{x}_h + kx + m_a(\ddot{u}_a\sin\theta_a + 2\dot{u}_a\dot{\theta}_a\cos\theta_a + (l_0 + u_a)\ddot{\theta}_a\cos\theta_a - (l_0 + u_a)\dot{\theta}_a^2\sin\theta_a) = 0 \tag{6}$$

$$m_a(l_0 + u_a)(2\dot{u}_a\dot{\theta}_a + l_0\ddot{\theta}_a + \ddot{x}_a\cos\theta_a + g\sin\theta_a) + c_{\theta_a}\dot{\theta}_a = 0 \tag{7}$$

$$m_a(\ddot{u}_a+\ddot{x}_h\ \sin\ \theta_a-(l_0+u_a)\dot{\theta}_a^2+g(1-\cos\ \theta_a))+k_au_a+c_a\dot{u}_a=0 \tag{8}$$

where the acceleration, velocity, and displacement of the hosting structure are denoted by $\ddot{x}_h$, $\dot{x}_h$, and x_h, respectively. The angular acceleration, the angular velocity, and the angle are denoted by $\ddot{\theta}_a$, $\dot{\theta}_a$, and θ_a, respectively. The acceleration, the velocity, and the displacement of the mass of the pendulum are denoted by $\ddot{u}_a$, $\dot{u}_a$, and u_a, respectively. By neglecting the damping coefficients, the resonance frequencies of the present system can be obtained as follows:

The *r*esonance frequency of the hosting structure:

$$f_{\text{hosting structure}}=\frac{1}{2\pi}\sqrt{\frac{k_h}{m_h}} \tag{9}$$

The resonance frequency of the PDVA mass−spring system:

$$f_{\text{mass−spring}}=\frac{1}{2\pi}\sqrt{\frac{k_a}{m_a}} \tag{10}$$

The resonance frequency of the PDVA pendulum system:

$$f_{\text{pendulum}}=\frac{1}{2\pi}\sqrt{\frac{g}{l_0+u_a(t)}} \tag{11}$$

One of the aspects of the present PDVA system is that the two resonance frequencies, f_{pendulum} and $f_{\text{mass-spring}}$, are utilized with the one mass-spring system. In addition, f_{pendulum} is changed continuously due to the change of $u_a(t)$. As the present PDVA has two resonance frequencies, the horizontal vibrations at the two resonance frequencies of the primary structure can be attenuated. In this simple one-dimensional system, the vertical vibration is ignored, but the above governing equations and the resonance frequencies suggest the possibility of the application of the PDVA system. The parameters in Eq. (12) are introduced to convert the second ODE to the first ODE.

$$\begin{bmatrix} x_h & \dot{x}_h & \theta_a & \dot{\theta}_a & u_a & \dot{u}_a \end{bmatrix}=\begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \end{bmatrix} \tag{12}$$

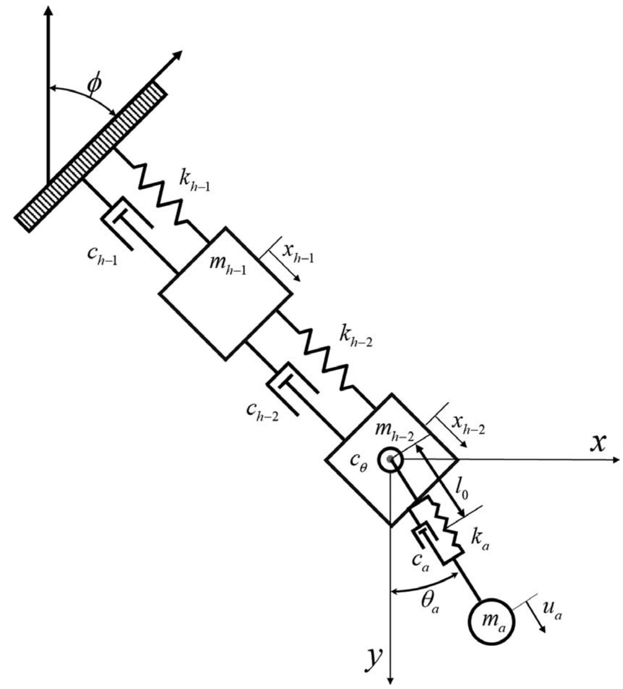

Fig. 3 The schematic diagram of the present pendulum dynamic vibration absorber attached the two degrees-of-freedom (2DOF) hosting structure (m_{h-1} and k_{h-1}: the mass and stiffness of hosting structure, m_{h-2} and k_{h-2}: the mass and stiffness of hosting structure, m_a and k_a: the mass and stiffness of present pendulum dynamic absorber, θ_a: the angle between the hosting structure and the present dynamic absorber, ϕ: the angle of the hosting structure, x_{h-1}, x_{h-2} and u_a: the displacements of the hosting structure and the dynamic absorber, c_{h-1}, c_{h-2}, c_θ, and c_a: the damping coefficients, l_0:the initial length of the dynamic absorber)

Then, the governing equations of the PDVA are formulated as follows:

$$\begin{bmatrix} \dot{x}_h \\ \ddot{x}_h \\ \dot{\theta}_a \\ \ddot{\theta}_a \\ \dot{u}_a \\ \ddot{u}_a \end{bmatrix}=\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix}=\begin{bmatrix} q_2 \\ \dfrac{-k_hq_1+m_h(\dot{q}_6\ \sin\ \theta+2q_6q_4\ \cos\ q_3-(l_0+q_5)\dot{q}_4\ \cos\ q_3+(l_0+q_5)q_4^2\ \sin\ q_3)+c_h\dot{q}_2}{(m_h+m_a)} \\ q_4 \\ \dfrac{(2q_6q_4-\dot{q}_2\ \cos\ q_3-g\ \sin\ q_3)}{m_a(l_0+q_5)}-\dfrac{c_\theta q_4}{m_a(l_0+q_5)^2} \\ q_6 \\ (\dot{q}_2\ \sin\ q_3-(l_0+q_5)q_4^2+g(1-\cos\ q_3))-\dfrac{k_aq_5}{m_a}-\dfrac{c_aq_6}{m_a} \end{bmatrix} \tag{13}$$

To numerically solve the governing equations with an ODE solver, Eq. (13) is numerically integrated with the ODE45 solver implemented in Matlab. It may be possible to linearize the above Eq. (13). As the present study aims to study the vibration reduction effect considering the finite angle changes of the pendulum, the linearization of the above governing equations is not pursued.

2.2 Application With Two Degrees-of-Freedom Hosting Structure. In this section, we extend the present PDVA system for the primary structure with two resonance frequencies in Fig. 3. The schematic diagram of the present pendulum dynamic vibration absorber attached to the 2DOF hosting structure is shown in Fig. 3. Note that the installation angle of the hosting structure is important as stated in Sec. 2.1. With a conventional spring-mass system or a pendulum system, we should install two vibration absorbers to attenuate the resonances of the system in Fig. 3. However, as the present PDVA system has the resonances of pendulum and mass-spring, it is sufficient to install just one PDVA system.

The governing equation of the system in Fig. 3 can be obtained by applying the Euler-Lagrange's principle.

$$\text{Kinetic energy } T = \frac{m_{h-1}\,\dot{x}_{h-1}^2}{2} + \frac{m_{h-2}\,\dot{x}_{h-2}^2}{2} + \frac{m_p(\dot{x}_p^2 + \dot{y}_p^2)}{2} \tag{14}$$

$$\begin{aligned}\text{Potential energy } V = {} & \frac{k_{h-1}x_{h-1}^2}{2} + \frac{k_{h-2}(x_{h-2}-x_{h-1})^2}{2} \\ & + \frac{k_p u_p^2}{2} + m_p g(l_0 + u_p - y_p)\end{aligned} \tag{15}$$

$$\text{Dissipation energy } F = \frac{c_{h-1}\,\dot{x}_{h-1}^2}{2} + \frac{c_{h-2}\,\dot{x}_{h-2}^2}{2} + \frac{c_\theta\,\dot{\theta}_p^2}{2} + \frac{c_p\,\dot{u}_p^2}{2} \tag{16}$$

The governing equations of the PDVA can be formulated as follows:

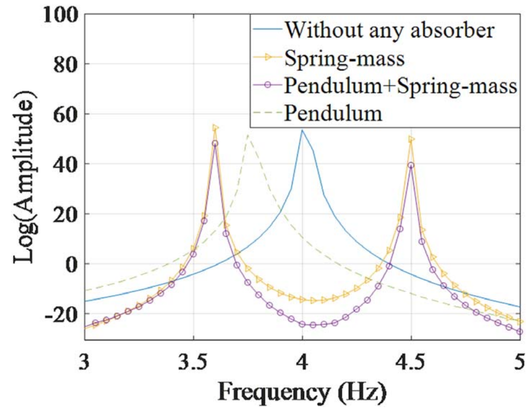

Fig. 4 Frequency response function of Fig. 2 ($m_h = 0.5103$ kg, $k_h = 325$ N/m, $m_a = 0.0153$ kg, $k_a = 10.75$ N/m, $l_0 = 0.0171$ m, $f_{\text{pendulum}} = f_{\text{mass-spring;}} = f_{\text{hosting structure}} = 4$ Hz)

$$\begin{bmatrix} \dot{x}_{h-1} \\ \ddot{x}_{h-1} \\ \dot{x}_{h-2} \\ \ddot{x}_{h-2} \\ \dot{\theta}_p \\ \ddot{\theta}_p \\ \dot{u}_p \\ \ddot{u}_p \end{bmatrix} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \\ \dot{q}_7 \\ \dot{q}_8 \end{bmatrix} = \begin{bmatrix} q_2 \\ \dfrac{1}{(m_1+m_3)}\begin{pmatrix} k_2 q_3 - m_3\,\dot{q}_4 - k_1 q_1 - k_2 q_1 + m_3 g\ \cos\ \phi - \\ m_3\,\dot{q}_8 \cos(\phi - q_5) + m_3 q_6^2\,\dot{q}_6 \cos(\phi - q_5) - \\ m_3 u\,\dot{q}_6 \sin(\phi - q_5) + m_3 l_0 q_2^6 \cos(\phi - q_5) - \\ 2m_3 \sin(\phi - q_5) q_6 q_8 - l m_3 \sin(\phi - q_5)\,\dot{q}_6 +\ \dot{c}_1 q_2 \end{pmatrix} \\ q_4 \\ \dfrac{1}{(m_2+m_3)}\begin{pmatrix} k_2 q_1 - m_3\,\dot{q}_2 - k_2 q_3 + m_3 g\ \cos\ \phi - \\ m_3 \cos(\phi - q_5) q_7\,\dot{q}_6 + m_3 l \cos(\phi - q_5) q_6^2 - \\ 2m_3 \sin(\phi - q_5) q_5 q_7 - ml \sin(\phi - q_5)\,\dot{q}_6 + c_2 q_4 \end{pmatrix} \\ q_6 \\ \dfrac{-m_3(l_0+q_7)}{m_3(l_0+q_7)(l_0+q_7+2q_8)}\left(g\ \sin\ q_5 + \sin(\phi - q_5)\,\dot{q}_2 + \sin(\phi - q_5)\,\dot{q}_4\right) \\ q_8 \\ \dfrac{1}{m_3}\begin{pmatrix} m_3\,\dot{q}_7 l - m_3 g - k_3 q_7 + m_3 g\ \cos\ q_5 + m_3\,\dot{q}_6 q_7 - \\ m_3 \cos(\phi - q_5)\,\dot{q}_2 - m_3\,\dot{q}_4 \cos(\phi - q_5) + c_3 q_8 \end{pmatrix} \end{bmatrix} \tag{17}$$

The above equations are again solved by the ODE45 solver.

3 Numerical and Experimental Studies

To prove the concept of the present PDVA, this section provides several numerical and experimental studies. The material properties in the numerical example are arbitrarily chosen to show the effectiveness of the present PDVA system.

3.1 The Spring-Mass System With Present PDVA. In the first example, the spring-mass system of Fig. 2 and Eq. (13) is considered. Without the loss of generality, the following initial condition is considered for the pendulum and spring-mass system

$$\begin{bmatrix} x_h & \dot{x}_h & \theta_a & \dot{\theta}_a & u_a & \dot{u}_a \end{bmatrix} = \begin{bmatrix} 0 & \dot{x}_h(0) & 0 & 0 & \dfrac{m_a g}{k_a} & 0 \end{bmatrix} \tag{18}$$

$$l_{steady-state} = l_0 + \frac{m_a g}{k_a} \tag{19}$$

where l_0 is the initial length of a spring and $\dot{x}_h(0)$ is the initial velocity of the hosting structure. Figure 4 shows the frequency response functions without any absorber, P, and s-m and pendulum. The frequency response functions with a pendulum (4 Hz for the resonance frequency of the pendulum and 4 Hz for the mass-spring system) are plotted in Fig. 4. As illustrated here, the vibration attenuation can be successfully achieved. The amplitude reduction at 4 Hz is significant but this can be achieved even with the vibration absorber with spring-mass or pendulum. Figure 5 shows the amplitudes in the time domain for the systems in Fig. 4. Figure 6 shows the amplitudes of the displacements of the pendulum and its effect on the resonance frequency of the pendulum. Note that the change of the displacement causes the changes in the resonance frequency.

3.2 Vibration Attenuations at Two Frequencies Using the Pre/Sent PDVA. To show the application of the PDVA absorber at two resonance frequencies, this second example considers the dynamic system of Fig. 3 with the material properties in Table 1. The first two resonance frequencies of the hosting structures are 1.923 Hz and 10.54 Hz. Then, it is pursued to attenuate the vibrations at these resonance frequencies simultaneously by installing one pendulum dynamic vibration absorber. As only one absorber suppresses the vibrations, it has advantages of reducing the space and the cost. To our best knowledge, it has not been attempted before this research. To achieve this, the resonance frequency of the pendulum, $(1/2\pi)\sqrt{(g/(l+u(t)))}$, is set to 1.923 Hz, and the resonance frequency of the spring-mass system, $(1/2\pi)\sqrt{(k/m)}$, is set to 10.54 Hz for the numerical simulation in Fig. 7. With 45 deg for

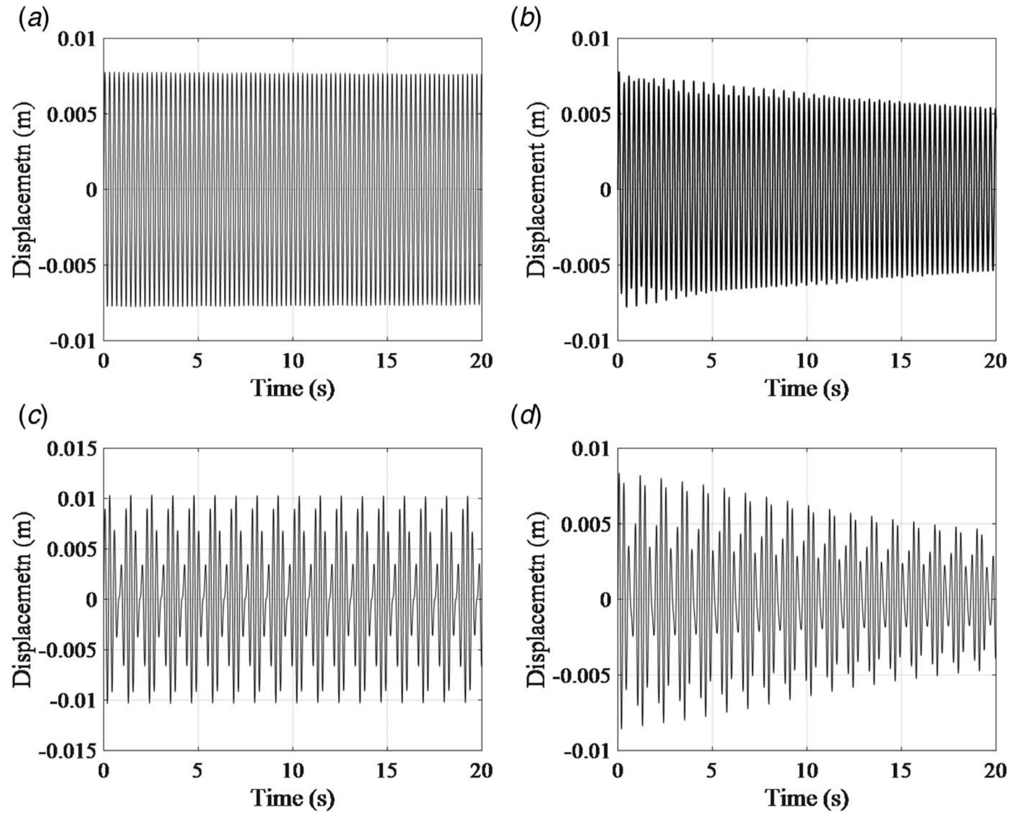

Fig. 5 Time response of Fig. 2: (*a*) without any absorber (no damping), (*b*) pendulum ($c_o = 10^{-6}$ N s/m), (*c*) spring-mass (no damping), and (*d*) pendulum + spring mass ($c_o = 10^{-6}$ N s/m)

ϕ, the initial simulation conditions are set as follows:

$$\begin{bmatrix} x_{h-1} & \dot{x}_{h-1} & x_{h-2} & \dot{x}_{h-2} & \theta & \dot{\theta} & u & \dot{u} \end{bmatrix} = \begin{bmatrix} 0 & \dot{x}_{h-1}(0) & 0 & 0 & 0 & 0 & \dfrac{m_a g}{k_a} & 0 \end{bmatrix} \tag{20}$$

Figure 7 shows the frequency responses with $\phi = 45$ deg at the masses. As illustrated, the resonance responses at the two frequencies are reduced with the help of the present PDVA. In addition, Fig. 8 tests the effect of the angle of the system. As the effective masses of the pendulum and the mass-spring system vary depending on the angle ϕ between the PDVA and the hosting structure, the angle ϕ affects the performance of the present PDVA. For example, Fig. 8 shows the response of x_{h-1} and x_{h-2} with some different angles of ϕ. When the present PDVA system is perpendicular to the direction of the movement of the hosting structure ($\phi = 0$ deg), the responses at the second frequency, $(1/2\pi)\sqrt{k/m}$, are not affected. This is because the effective mass of the PDVA is zero. On the other hand, with 90 deg for ϕ, i.e., when the PDVA is parallel to the hosting structure, the effective mass of the pendulum is zero and the responses are little affected at the first frequency, $(1/2\pi)\sqrt{(g/(l + u(t)))}$.

3.3 Experiment. In the previous subsections, the numerical studies were conducted to show the characteristics and the advantages of the present pendulum dynamic absorber. To verify experimentally, this section conducts two experiments with the experiment setup (impact hammer, NI-9234 DAQ device, and an

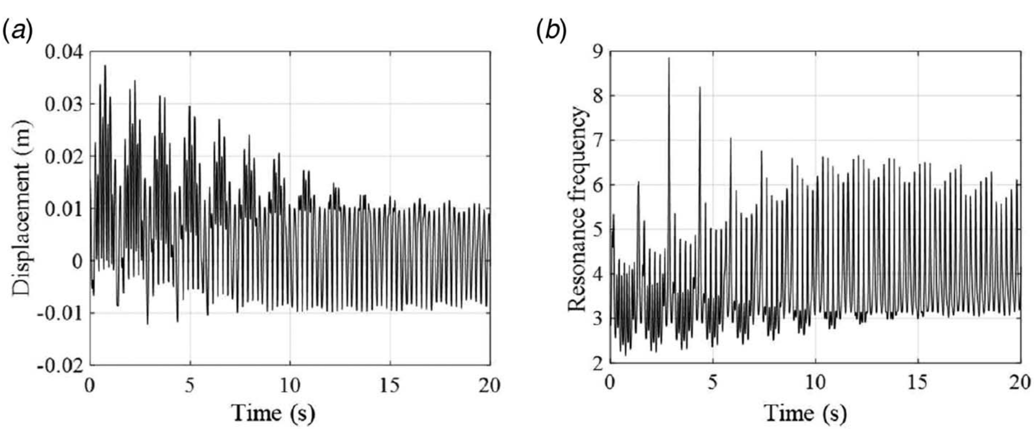

Fig. 6 Change of resonance frequency of Fig. 5(*d*), $\frac{1}{2\pi}\sqrt{\frac{g}{l+u(t)}}$

Table 1 Parameters of the PDVA and hosting structure at case 2

G	9.81 m/s^2	m_a	0.1 kg
m_{h-1}	2.5 kg	k_a	420 N/m
k_{h-1}	600 N/m	c_a	0.1 N s/m
c_{h-1}	1.2 N s/m	f_{h-1}	1.923 Hz
m_{h-2}	1.5 kg	f_{h-2}	10.54 Hz
k_{h-2}	4000 N/m	f_{a-p}	1.923 Hz
c_{h-2}	1.2 N s/m	f_{a-mk}	10.54 Hz
l_0	0.066 m	φ	0 deg:15 deg:90 deg

accelerometer) in Fig. 9. To substitute the spring-mass system in Figs. 2 and 3, a cantilever beam that can be regarded as a distributed spring-mass system is employed.

The first- and the second-resonance frequencies of the cantilever beam in Fig. 9 are $f_1 = (1.875/2\pi)\sqrt{(EI/\rho AL^4)}$ Hz and $f_2 = (4.694^2/2\pi)\sqrt{(EI/\rho AL^4)}$ Hz where Young's modulus, the moment of inertia, the area, the length, and the density are denoted by E, I, A, L, and ρ, respectively.

Experiment 1: The beam vibration with the pendulum

Figure 10 shows the schematic diagram of the first experiment which is corresponding to the first numerical simulation. This experiment is aimed to show that it is possible to attenuate the structural vibration with the pendulum dynamic vibration absorber by tuning the pendulum frequency and the spring-mass frequency to the resonance frequency of the beam as studied in Fig. 4.To excite the beam, the impact hammer is employed to apply an impact force at point A in Fig. 10 and the acceleration at point B is measured with the accelerometer. The frequency response functions with and without the PDVA are plotted in Fig. 11. As expected, it is possible to attenuate the vibration successfully with more than 20 dB. As the resonance frequency of the absorber is matched with the resonance frequency of the hosting structure, the significant reduction is achievable. However, the mutual effects of the pendulum and the spring-mass are accumulated, and the significant vibration reduction can be achievable. To our best knowledge, it is rare to use one vibration absorb system to realize the effect of spring-mass and pendulum simultaneously. Furthermore, as the length of the spring-mass system is varying during the vibration, the vibration attenuations can be achievable at a wider frequency range.

Experiment 2: The beam vibrations at two frequencies

To show the advantage of the present PDVA system at two frequencies which is corresponding to the second numerical example, the experiment in Fig. 12 is considered. This experiment aims to attenuate the first- and the second-bending modes (1.5 Hz and 9.7 Hz) simultaneously only with the one simple PDVA system. Note that the PDVA system is made with one spring and

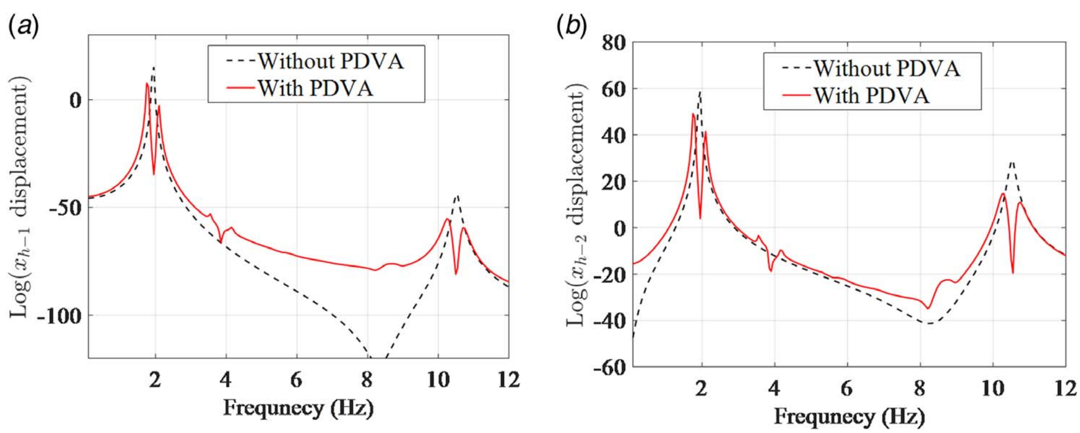

Fig. 7 The frequency response functions of Fig. 3 ($\phi = 45$ deg): (a) the response of the hosting structure x_{h-1} and (b) the response of the hosting structure x_{h-2}

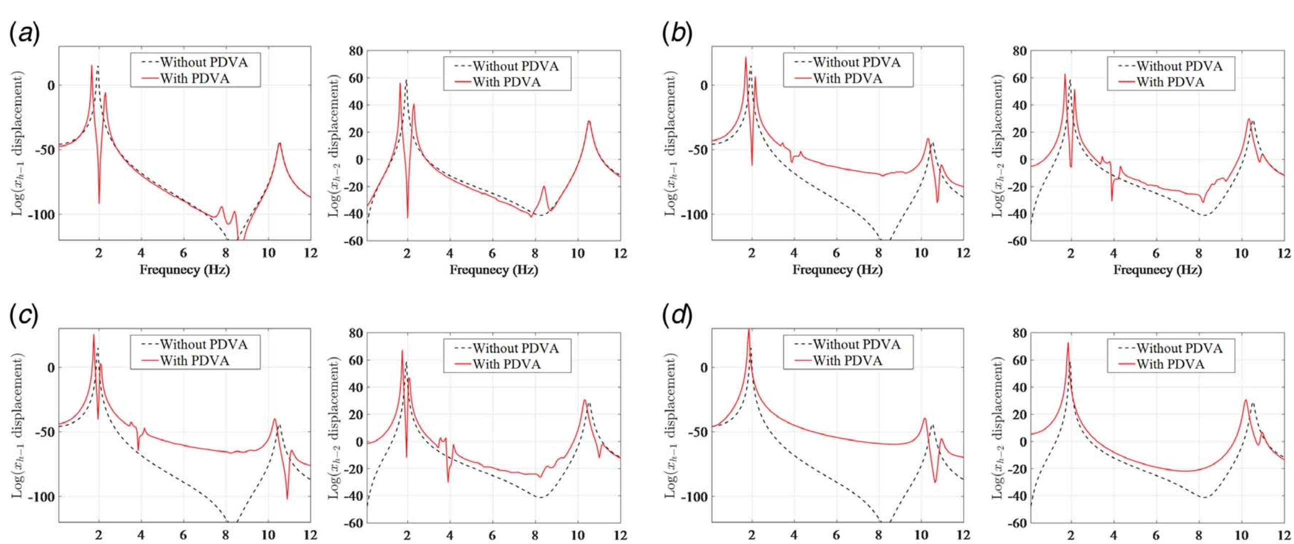

Fig. 8 The effects of the installation angle: (a) $\phi = 0$ deg, (b) $\phi = 30$ deg, (c) $\phi = 60$ deg, and (d) $\phi = 90$ deg

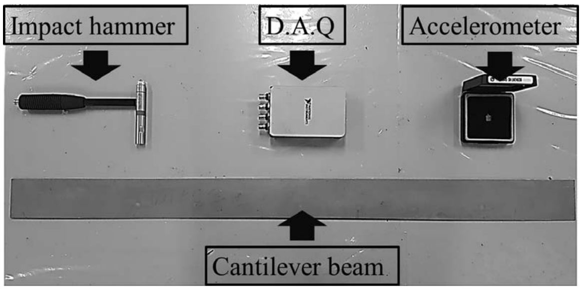

Fig. 9 Experiment setup (steel beam, thickness = 2 mm, E = 200 GPa, width = 5 cm, density = 7800 kg/m^3)

one mass. In this experiment, we allow the swing of the spring and the mass to utilize the effect of the pendulum in addition to the spring-mass system. To do this, the impact force is applied at point A horizontally, and this force is sufficiently large enough to induce the swing motion of the spring with varying its length. The spring constant, the length, and the mass are tuned a summarized in Table 2 to make the pendulum frequency 1.5 Hz and the spring-mass frequency 10 Hz (it was impossible to perfectly match in the experiment). As the systems are different, it is impossible to have the same responses to the simulation and the experiment. However, we intended to predict the responses of the first experiment with the first numerical simulation in Fig. 4 and the second experiment with the second numerical simulation in Fig. 7.

Figure 13 shows the frequency responses at the x-direction of the B point with and without the PDVA. As observed in this experiment, the present PDVA is effective in reducing the accelerations at the two resonance frequencies. Note that the initial angle of the absorber is 0 deg. However as we consider the finite angle of the dynamic absorber, it is possible to obtain the response reduction in the second resonance. We expect that the present PDVA is efficient in terms of space and cost.

4 Conclusions

This research presents the PDVA consisting of a spring and a mass to attenuate structural vibrations of hosting structure at multiple frequencies. Commonly several dynamic absorbers are attached to hosting structure to attenuate vibrations at multiple frequencies.

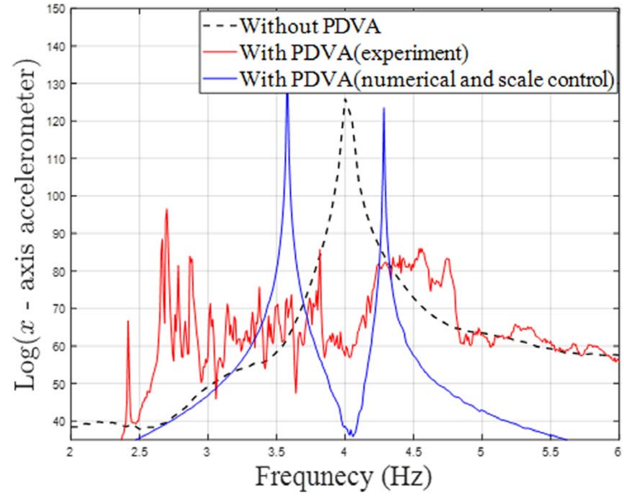

Fig. 11 Acceleration at the point B with and without the PDVA

Inevitably it increases the total mass and the cost and causes the installation limitations for some circumstances. Therefore the reductions in the number of vibration absorbers are one of the important topics in engineering. To contribute to this research field, the present research proposes to apply the pendulum dynamic absorber possessing two resonance vibrations, i.e., the resonance of mass-spring and the resonance of a pendulum. The present PDVA being composed of a spring and a mass, the structural vibrations at the two resonance frequencies, i.e., the square root of mass over stiffness and the square root of gravidity over length, can be simultaneously applied to attenuate the structural vibrations of hosting structure. The length of the spring is subject to be varied due to the resonance vibration of mass-spring, the resonance frequency of pendulum can vary. One of the aspects we should care about is that the effective masses of the spring-mass and the pendulum can be different depending on the motions of the hosting structure. It is observed that it is better to pose the PDVA system obliquely to the main vibrations of hosting structure to maximize the energy dissipations in the radial direction for the mass-spring system and the circumferential direction for the pendulum system. The change of the spring length and its associated

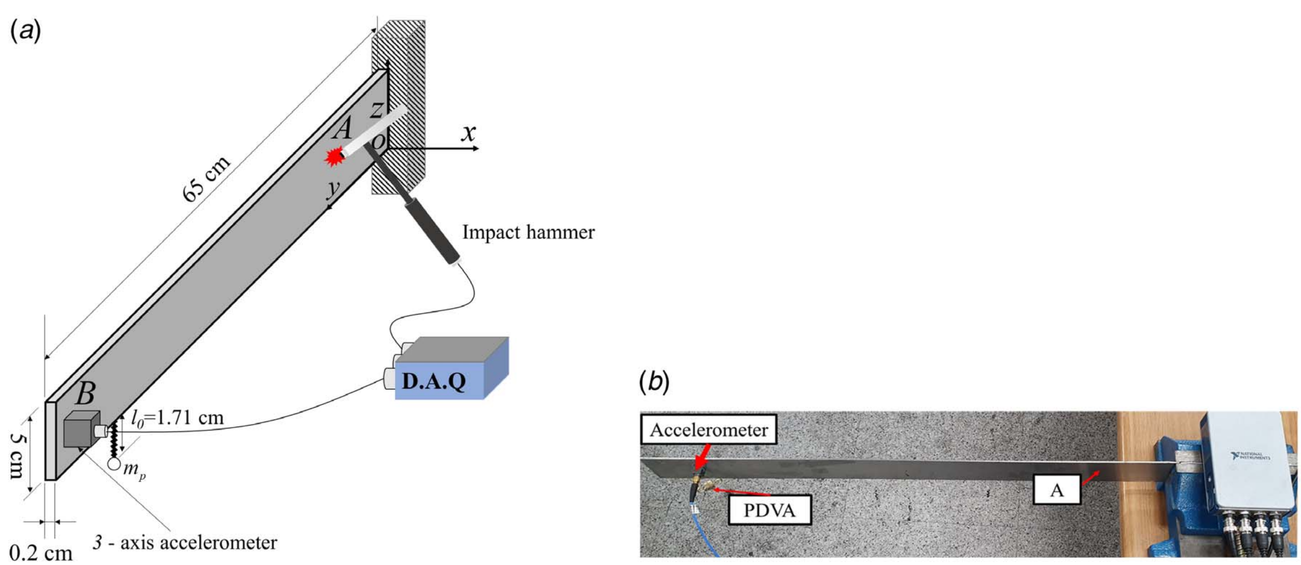

Fig. 10 Experiment 1: the bending vibration of the beam (L = 0.65 m) (the first-bending mode of the beam: 4.0 Hz, the pendulum frequency: 4.0 Hz, and the spring-mass frequency: 4.0 Hz)

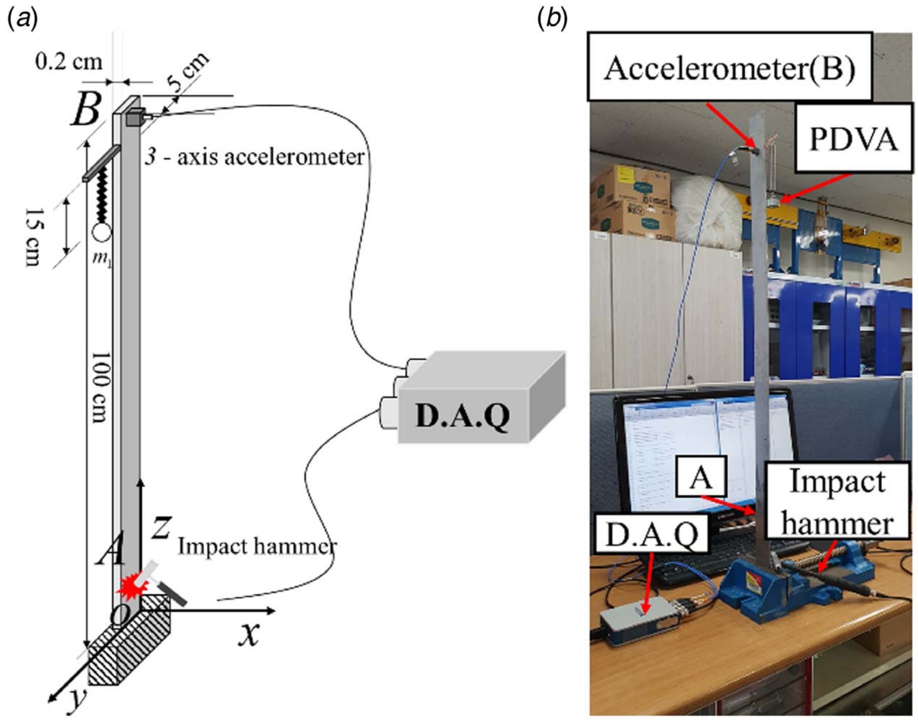

Fig. 12 Experiment 2: the bending vibration of the beam (L = 1 m). (the first-bending mode of the beam: 1.5 Hz and the second-bending mode of the beam: 9.7 Hz, and the pendulum frequency: 1.5 Hz and the spring-mass frequency: 10 Hz)

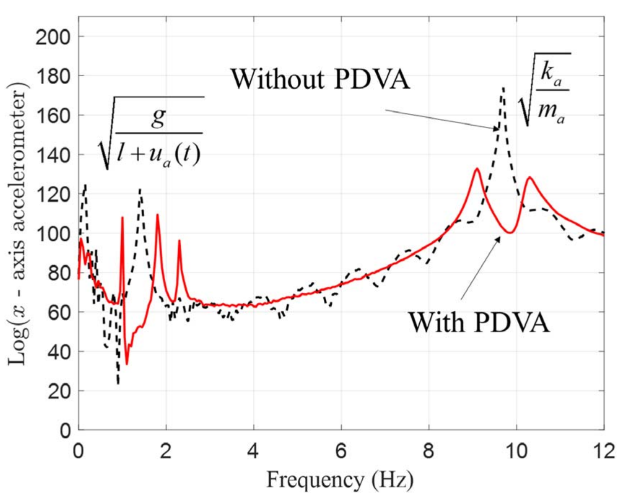

Fig. 13 Acceleration at the point B with and without the PDVA

Table 2 Geometry parameters and material properties for the second experiment

Spring stiffness	240 N/m	Beam width	0.05 m
Mass of PDVA	0.050 kg	Beam thickness	0.002 m
Initial length of PDVA	0.15 m	Steel density	7800 kg/m^3
Beam length	1.00 m	Young's modulus	200 GPa

variations of the pendulum resonance frequency allows us to the wave attenuation at a wider frequency range which is important from an engineering point of view. When equating the two resonance frequencies of the present PDVA, it is also possible to achieve an accumulated vibration attenuation. In addition, the vibrations of hosting structure at two distinct frequencies can be suppressed by the present PDVA by matching the resonance frequencies of the PDVA to the resonance frequencies of the hosting structure. Some numerical simulations and experiments are also carried out to prove the concept of the present PDVA system. For future research, we expect that the present PDVA can be applied to the applications of materials or metasurfaces.

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