



## Research Paper

## Investigation on numerical analysis and mechanics experiments for topology optimization of functionally graded lattice structure

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## ABSTRACT

A physical driven method for constructing functionally graded lattice structure is proposed by extending the conventional Solid Isotropic Material with Penalization (SIMP) method with linear buckling load factors and total volume constraints rather than introducing local volume constraints. It can efficiently and effectively construct functionally graded lattice structure by using the SIMP material model for topology optimization with different buckling load factors according to the linearized buckling criteria and the same volume fraction, which greatly improved both the stability and stiffness of the optimal structure. Furthermore, the optimized structures of the illustrative cases are manufactured by means of the fused deposition modeling. Both the numerical analysis and mechanics experiments are further conducted to illustrate the effectiveness of the proposed method compared with the structural topology optimization with SIMP method and the infill structure. To the best of our knowledge, it is the first time to validate the topology optimization of functionally graded lattice structure via both numerical analysis and mechanics experiments.

## 1. Introduction

Functionally graded lattice structure has been widely used in industry due to its excellent performance on mechanical characteristic and lightweight property, especially civil engineering, aerospace, railway infrastructure. Due to that the pattern of the functionally graded lattice structure is complex and variable, so that it is difficult to be manufactured with conventional mechanical manufacturing techniques such as milling, casting, and extrusion. Thanks to the significant development in additive manufacturing, it makes the fabrication of complex topological geometry possible, simple and feasible. Therefore, many researchers are devoted to designing the topological geometry and validating the mechanics characteristic of the optimal structure with functionally graded lattice [1–4].

Researchers have done a lot of studies on the functionally graded material structure (FGMs), it performs the typical physical properties through reasonably distributes material over the design domain, of which the physical properties gradually change with position. Zghal et al. detailed finite element modeling and simulations of the functionally graded material structures from bending, vibrational behavior and mechanics characteristic perspective [5–8]. Tahir et al. investigated the

wave propagation of a ceramic-metal functionally graded sandwich plate [9,10]. Merazka and Mudhaffar et al. studied the bending response of the functionally graded plate under hygrothermo-mechanical load [11,12]. Hachemi et al. analyzed the bending behavior of functionally graded plates using a new refined shear deformation theory and the concept of the neutral surface position [13]. Bakoura et al. introduced a higher shear deformation theory to analyze buckling response of simply-support functionally graded plates [14]. Zghal et al. investigated buckling and post-buckling behaviors of functionally graded material structures, who mainly consider buckling response under various mechanical loadings while not the structure designing [15,16]. It can be found that the traditional methods mainly focus on the simulation or experiment of the functionally graded structures, not straight for design and optimization, and the multiple attempts of experiment will cost much time and efforts. Thus, a convenient pipeline for constructing functionally graded structure should be further investigated.

The functionally graded lattice structure is a variant of FGMs. The spatial patterns of functionally graded lattice structures are consisted of numerous trusses, which improve the stability of the structure. In the recent years, various topology optimization methods, including homogenization method [17,18], SIMP [19,20], Level Set method [21,22],

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Bidirectional Evolutionary Structural Optimization [23,24], and Moving Morphable Components method [25,26], have been developed to derive functionally graded lattice structures. Pattern repetition and local volume constraints are commonly used approaches to construct lattice structures [27]. The former one, pattern repetition applies the predefined patterns to the full domain. Bendsøe and Kikuchi firstly introduced microstructure into structural design, and used the material homogenization of parameterized lattice cells to fulfill the optimal structure [28]. Fu et al. introduced a multiscale level set topology optimization method to produce shell-coated macro structures, and the uniform micro structure is optimized to achieve shell-infill structures [29]. Chu et al. constructed multifarious macrostructures to fill inner base part of the coated structure which is determined by the parametric level set method [30]. The latter one employs the local volume constraints to limit the volume fraction of the prescribed filter zoom. Wu et al. added local volume constraints to SIMP for generating bone-like porous structures while structure stiffness decreased when compared with standard SIMP structure [31]. Following with the previous work, Literature [32] reported a strategy of using multiple smooth and projection method to design shell-infill lattice structure. Liu introduced two-filed-based parameterization and localized volume constraint for designing the self-supporting infill structures [33].

Nevertheless, the lattice structure constructed through the previous methods are composed of numerous long and slender features which are prone to buckling failure under compression, thus further developments of lattice structure have been proposed to improve the stability against buckling failure [34]. Yi et al. presented a topology optimization method with buckling constraints by minimizing structure compliance to produce functionally graded lattice structure, which achieves high spatial lattice variations and great improvement of the structure stability at the cost of increased compliance [35]. Ferrari et al. revisited the topology optimization with buckling constraints and presented various lower bounds of buckling load factor to build lattice pattern for improving structure performance, nevertheless the computation effort of the optimization problem is fairly expensive [36]. Clausen et al. addressed that the infill-based components in coated structures shows excellent performance against buckling failure compared with the conventional topology optimization approach, while it is optimized with stiffness rather than stability [37–39]. There are still a lot of research papers in the field of buckling analysis for functionally graded lattice structure, we cannot review all the papers here, the interesting readers can refer to literatures [15,16,40,41]. However, both the periodic patterns and local volume constraints are restrictive for searching the optimal results in feasible solution space and limited the structure performance against buckling failures [27].

Overall, anyone who analyzes the functionally graded lattice structure by experimental method mainly focus on the manufacturing rather than optimal design, while another group designs topological structures by optimization theory mainly focus on numerical analysis without mechanics experiments. Even though the progress on studying infill structure construction has been largely promoted, the practical mechanical characteristic of the optimal structure with buckling constraints should be further investigated and discussed, especially the stiffness, strength, and stability performance. The inadequate investigation on constructing functionally graded lattice structure urge this work which is the first time to validate the performance of functionally graded lattice structure by both numerical analysis and mechanics experiments.

To construct functionally graded lattice structure while considering structure stiffness and stability, rather than introducing the periodic patterns or local volume constraints, we present a topology optimization with buckling constraints based on Solid Isotropic Material with Penalization (SIMP) method. Different buckling load factors that following the linear buckling criteria are employed to generate lattice structures. Based on numerical analysis of the optimized structure, the compressive mechanics experiments are conducted to investigate the

mechanical property, which acts in accordance with the numerical analysis, and demonstrated the performance on the proposed method with conventional SIMP method and the infill structure.

The main **contributions** of this work can be summarized as follows:

1. A physical driven method for constructing functionally graded lattice structure is proposed. To the best of our knowledge, it is the first time to validate the topology optimization of functionally graded lattice structure via both numerical analysis and mechanics experiments.
2. The proposed method extends conventional SIMP model with linear buckling load factors and total volume constraints rather than introducing local volume constraints, which improves both the stiffness and stability.
3. Both the numerical analysis and mechanics experiments validate the advantage of the proposed method compared with standard SIMP and infill methods.

The remaining of this paper is organized as following. The mathematical formulation and analysis process are provided in Section 2. In Section 3, numerical analysis and mechanics experiments are conducted to illustrate the effectiveness of the proposed method. Finally, the conclusion and future work are presented in Section 4.

## 2. Optimization problem formulation

### 2.1. Material model

Given the discretized design domain  $\Omega$ , our goal is to maximize the global stiffness by an approximate material distribution, which the structure is filled with solid and void elements. However, integer programming problems with large-scale parameters are difficult to be solved, and even worse to utilize efficient gradient-based optimization techniques. Consequently, the optimization problem is relaxed by introducing a design variable  $\Phi$  which continuously vary from 0 to 1. Then Helmholtz-type differential equations with homogeneous Neumann boundary conditions are introduced as a density filter method to avoid checkerboard patterns and smooth the density of the optimized geometric structure [42,43], which is shown as following:

$$-r^2 \Delta \bar{\Phi} + \bar{\Phi} = \Phi \quad (1)$$

where  $\Phi$  is the design variable and the filtered density is described by  $\bar{\Phi}$ . The parameter  $r$ , a similar role as  $R$  in the classical filter method [44], controls the radius of filter zooms. The relation between  $r$  and the classical filter radius  $R$  can be describe as:

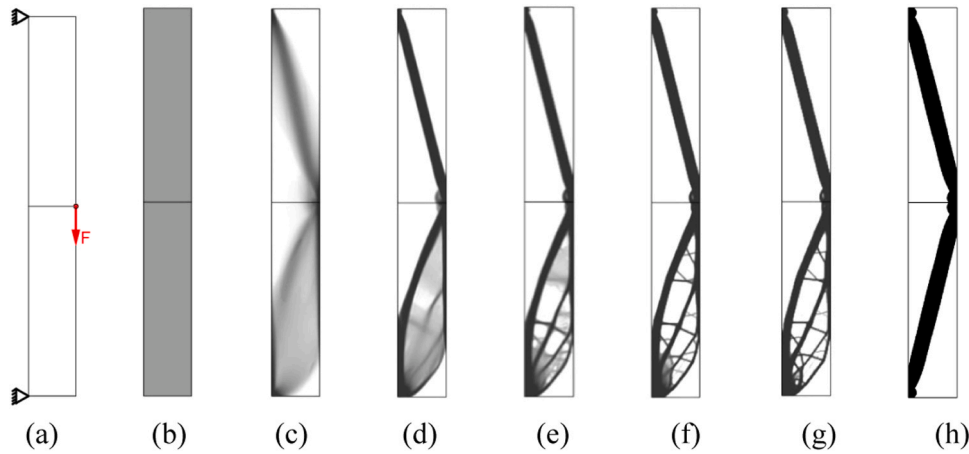
$$r = \frac{R}{2\sqrt{3}} \quad (2)$$

The filtering technique indeed solve the check board problems while gray the narrow band between solid and void elements [21,45], thus the Heaviside projection is used to alleviate such situations and speed converging process which is shown as follows:

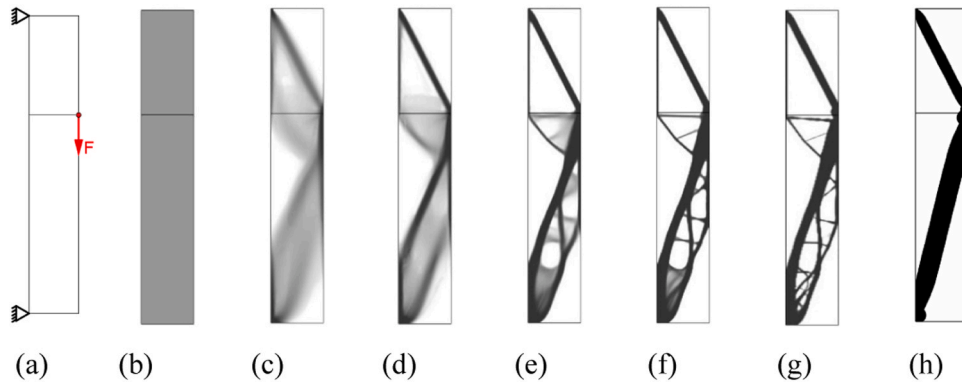
$$\rho = H(\bar{\Phi}) = \begin{cases} 0 & \bar{\Phi} < -\delta \\ 0.75 \left( \frac{\bar{\Phi} - \bar{\Phi}^3}{\delta - 3\delta^3} \right) + 0.25 & -\delta \leq \bar{\Phi} < \delta \\ 1 & \bar{\Phi} \geq \delta \end{cases} \quad (3)$$

The parameter  $\delta$  controls the sharpness of the projection function and there we assign  $\delta$  with 0.75 and gradually decrease the value until reach the lower bound 0.25. The adjustment of the parameter  $\delta$  allows the material distribute reasonably and guarantees the quick convergence at the optimal point.

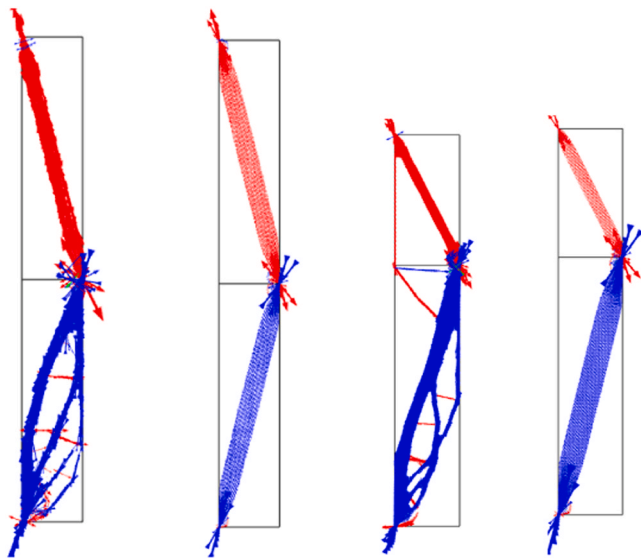
The material properties following Solid Isotropic Material with Penalization principle is considered in this paper. The elasticity modulus of finite element meshes with continuous material distribution  $\rho_e$  can be



**Fig. 1.** The configuration of symmetrical design area and the optimal structures of the considered example, (a) the design domain and its boundary conditions, (b)–(g) the iterative process of optimization results obtained by proposed method, (h) the optimal structure with conventional SIMP method.



**Fig. 2.** The configuration of unsymmetrical design domain and the optimization results of the considered example, (a) the design domain and its boundary conditions, (b)–(g) converge history of the functionally graded lattice structure, (h) the optimal structure with conventional SIMP method.



**Fig. 3.** Comparison of the principal stress of optimized structures of Figs. 1 and 2.

expressed as:

$$E(\rho_e) = E_{min} + \rho_e^p (E_0 - E_{min}) \quad (4)$$

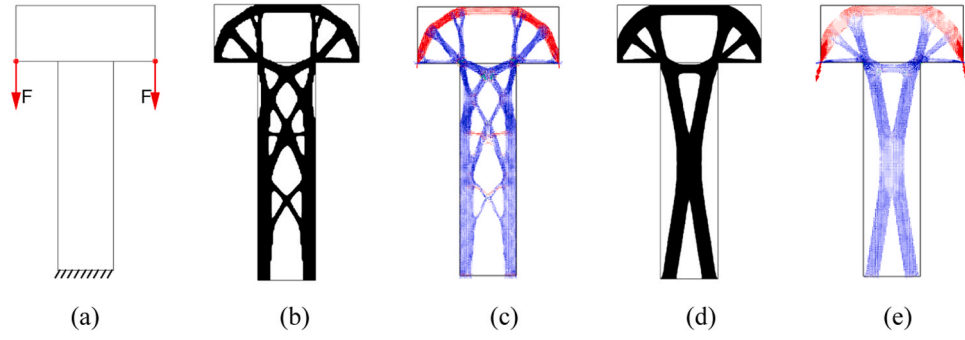
where  $E_0$  is the Young's modulus of solid material and  $E_{min}$  is the stiffness of the void element which is assigned a fairly small value to avoid matrix singular in the process of finite element analysis.  $p$  is the penalization power to guarantee the density of the element converge to 0 or 1 as much as possible ( $p \geq 3$  is usually required).

## 2.2. Buckling load factors computation

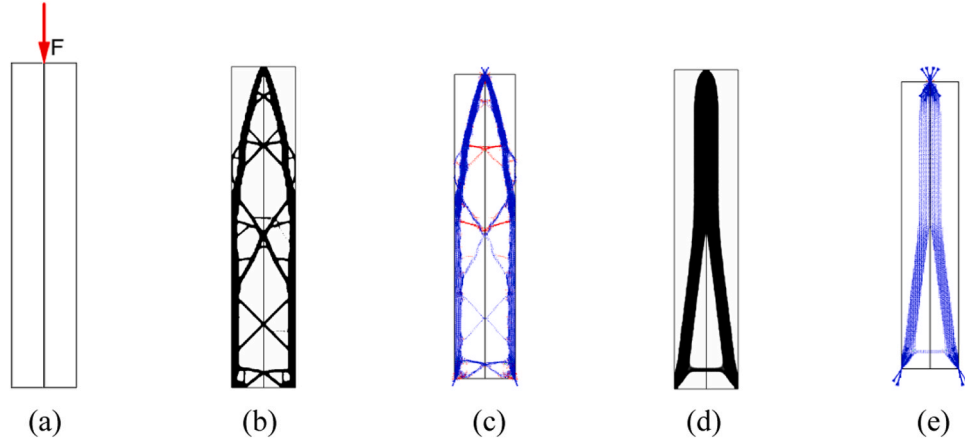
Buckling analysis is mainly used to study the stability of the structure under a specific load and determine the critical load for structural instability. There are two major buckling analysis methods: linear and non-linear buckling analysis [46,47]. We address linearized buckling analysis problem which can be set as:

$$(K + \lambda_j K_\sigma) \varphi_j = 0 \quad (5)$$

where  $K$  is the global stiffness matrix and the geometric stiffness matrix is described by  $K_\sigma$ , and  $\lambda_j$  and  $\varphi_j$  are buckling load factor and buckling mode vectors respectively. As the geometric matrix is the function of stress, which means  $K_\sigma$  is indefinite, thus the eigenvalues computed by equation may both have positive and negative value, normally only the positive buckling load factors are considered for buckling mode analysis.



**Fig. 4.** The configuration of the T-shaped example. The design domain and its boundary conditions (a), the optimal results (b) (d) and the distribution of the principal stress (c) (e) by the proposed method and SIMP.



**Fig. 5.** The configuration of the illustrative examples. The design domain and its boundary conditions (a), the optimal results (b) (d) and the distribution of the principal stress (c) (e) by the proposed method and SIMP.

### 2.3. Problem formulation

Based on the stated material interpolation model, the mathematical formulation of topology optimization with buckling constraints can be expressed as follows:

$$\begin{aligned}
 & \text{find } \phi = \{\phi_1, \phi_2, \dots, \phi_n\} \\
 & \min C = F^T U \\
 & \text{s.t. } V(x)/V_0 \leq f_0 \\
 & F = KU = \sum_{i=1}^n E(\rho_i) k_0 u_i \\
 & \lambda_i \geq \lambda^*
 \end{aligned} \quad (6)$$

here  $f_0$  is prescribed volume fraction and  $\lambda^*$  is prescribed minimum buckling load factor, and  $k_0$  is the unit Young's modulus of element stiffness matrix, and  $u_i$  is the displacement vector of the element  $i$ . The stated constrained optimization problem can be solved by gradient-based method, typically analyzing the sensitivity of objective function, constraints and intermediate variables with respect to design variables.

### 2.4. Sensitivity analysis

According to the statement of the problem formulation, we briefly describe the sensitivity analysis of the formulation.

The sensitivity of intermediate variable  $\bar{\phi}$ , the smoothed density field, with respect to design variable can be calculated as Eq. (7), where

$N$  is shape interpolation function for finite element analysis.

$$\frac{\partial \bar{\phi}}{\partial \phi} = \left[ \sum \int (-\nabla N^T R^2 \nabla N + N^T N) d\Omega_e \right]^{-1} \cdot \left[ \sum \int N d\Omega_e \right] \quad (7)$$

The sensitivity of projection function with respect to smoothed design variable can be derived as following, where  $\delta$  is the narrow band of the projection function.

$$h(\bar{\phi}) = \frac{\partial \rho}{\partial \bar{\phi}} = \begin{cases} \frac{3}{4\delta} \left( 1 - \frac{\bar{\phi}^2}{\delta^2} \right) & |\bar{\phi}| \leq \delta \\ 0 & |\bar{\phi}| > \delta \end{cases} \quad (8)$$

Then, considering the sensitivity of intermediate variables with respect to design variables have derived, the objective compliance with respect to design variables can be easily calculated by using chain rules which is shown as,

$$\frac{dC}{d\phi} = \frac{\partial C}{\partial \rho} \frac{\partial \rho}{\partial \bar{\phi}} \frac{\partial \bar{\phi}}{\partial \phi} \quad (9)$$

As we can see from the objective function that the external force which applied on each element are the function of design variables, by using adjoint method that we could derive the first differential operation  $\partial C / \partial \rho$  which is shown as Eq. (10),

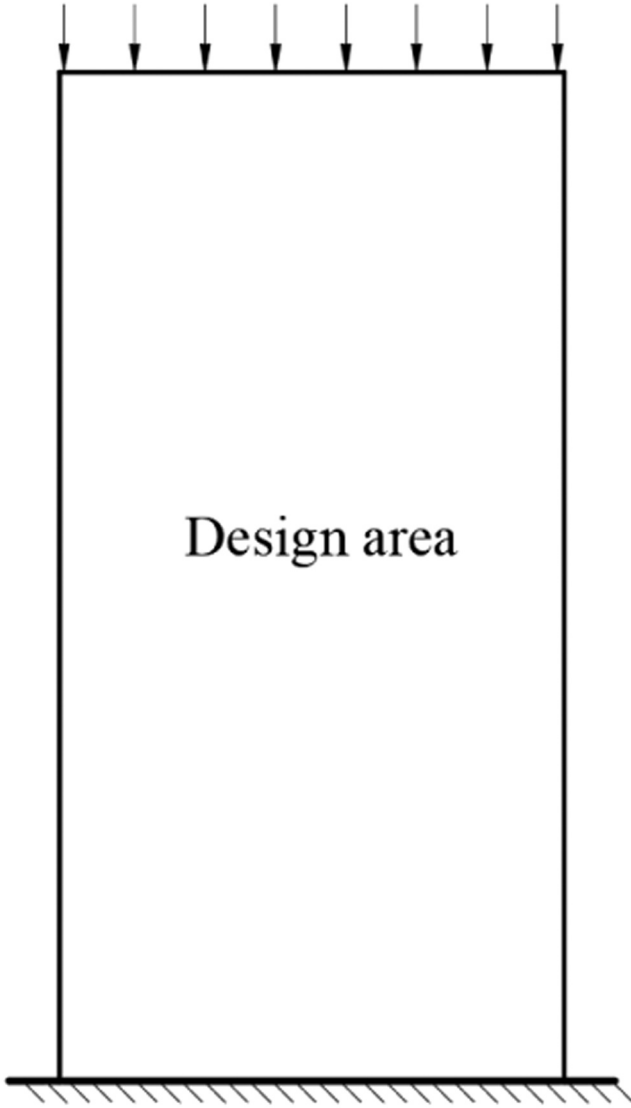


Fig. 6. The design domain and the boundary conditions of the structure.

$$\frac{\partial C}{\partial \rho} = \sum_{i=1}^n u_i^T p \rho_i^{p-1} E_0 k_0 u_i \quad (10)$$

Combining with Eqs. (7), (8) and (10), the sensitivity of objective function can be formulated as following:

$$\frac{dC}{d\phi} = \sum_{i=1}^n u_i^T p \rho_i^{p-1} E_0 k_0 u_i \cdot h(\bar{\phi}) \cdot \left[ \sum \int (-\nabla N^T R^2 \nabla N + N^T N) d\Omega_e \right]^{-1} \cdot \left[ \sum \int N d\Omega_e \right] \quad (11)$$

As for the sensitivity of total volume constraint in respect to design variables, it can be derived as Eq. (12),

$$\begin{aligned} \frac{dV}{d\phi} &= \frac{\partial V}{\partial \rho} \frac{\partial \rho}{\partial \bar{\phi}} \frac{\partial \bar{\phi}}{\partial \phi} \\ &= h(\bar{\phi}) \cdot \left[ \sum \int (-\nabla N^T R^2 \nabla N + N^T N) d\Omega_e \right]^{-1} \cdot \left[ \sum \int N d\Omega_e \right] \end{aligned} \quad (12)$$

Computing the sensitivity of buckling load factors is quite trivial and is not straightforward to compute, therefore we follow the study of literature [48] and briefly describe the results here, refer to our previous work [37] for the detailed derivation.

$$\frac{\partial \lambda_j}{\partial \phi} = \frac{\partial \lambda_j}{\partial \rho} \frac{\partial \rho}{\partial \bar{\phi}} \frac{\partial \bar{\phi}}{\partial \phi} = \frac{\varphi_j^T p \rho^{p-1} E_0 k_0 - \mu^T \frac{\partial E_{ge}}{\partial \rho} B d + \varpi^T B^T \frac{\partial E}{\partial \rho} B d}{\varphi_j^T K_\sigma \varphi_j} \cdot h(\bar{\phi}) \quad (13)$$

$$\cdot \left[ \sum \int (-\nabla N^T R^2 \nabla N + N^T N) d\Omega_e \right]^{-1} \cdot \left[ \sum \int N d\Omega_e \right]$$

Where  $\mu$  and  $\varpi$  are the adjoint variables shown in Eqs. (14), (15), and  $E_{ge}$  is the geometric Young's modules, the strain-displacement matrix is denoted by B.

$$\mu^T = -\lambda_j \varphi_j^T \frac{\partial K_\sigma}{\partial \sigma} \varphi_j \quad (14)$$

$$\varpi^T = (\mu^T E_{ge} B) (B^T E B)^{-1} \quad (15)$$

### 3. Experiments and analysis

#### 3.1. Numerical examples

##### 3.1.1. Example 1

To demonstrate the effectiveness of the proposed method to construct functionally graded lattice structure, we follow the study of [38] and present numerical examples for examination. The configuration of the first example is shown as the Fig. 1(a), the height and width of the I shape design region are 2 and 0.25 respectively, the top and bottom of the left side points are fixed, and a unit concentrate load is applied on the middle right side. The design domain is discretized into  $400 \times 50$  elements, and Young's modulus and Poisson ratio are set as 1 and 0.3 respectively, the filter radius is set as the element size and the upper bound of volume fraction is set to 0.3, and the buckling constraints is 2.5. Fig. 1(b)–(g) exhibit the functionally graded lattice patterns at the iterative process and Fig. 1(h) gives the optimal structure produced by conventional SIMP method. In contrast to SIMP structure, the functionally graded lattice structure consists of two main patterns, the upper structure presents strut pattern and suffers from tensile stress, and the lower one filled with truss-like structures where suffering from compressive stress which improves the resistance of the structure buckling compared with the SIMP structure that presents a symmetric pattern.

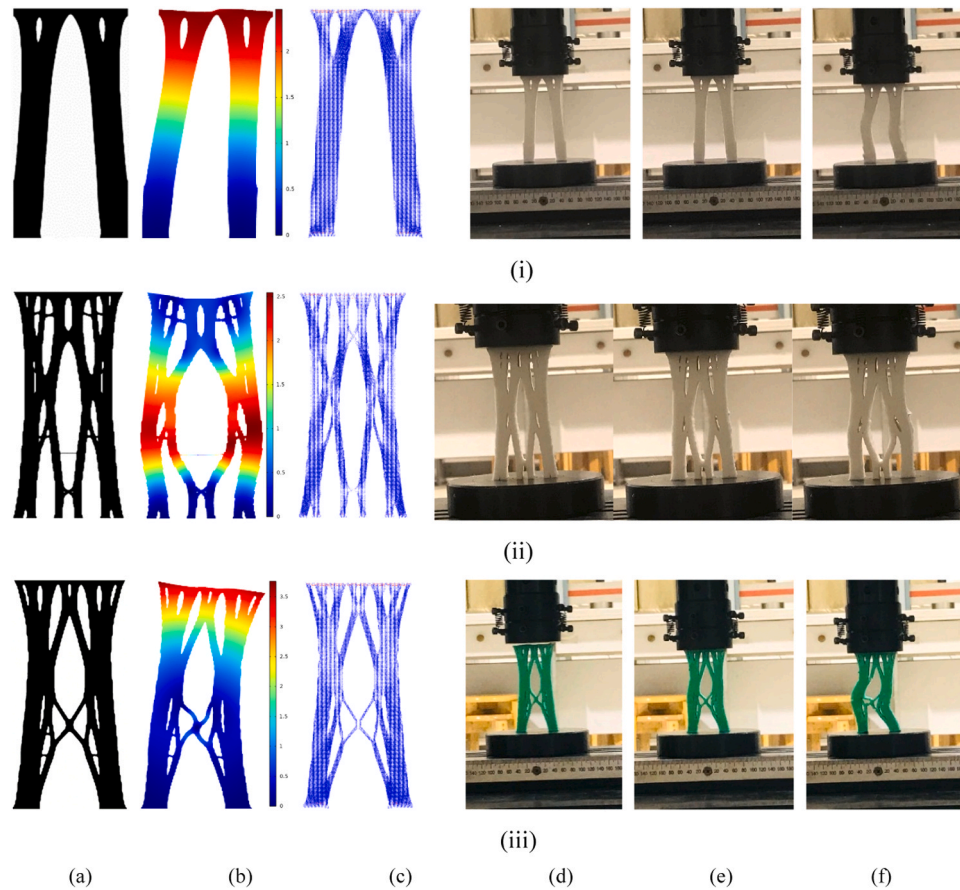
Furthermore, the unsymmetrical I shape design domain is adopted to illustrate the universal effect as shown in Fig. 2, the height and width of the design region, which is discretized into  $300 \times 50$  equally-sized elements. The proposed method performs desirable effect to construct

functionally graded lattice structure, which the upper part showing as the strut and lattice patterns generated in the compressive part. From the distribution of the tensile and compressive stress of the optimized structures shows in Fig. 3, we notice that the inner truss-like structures of functionally graded lattice structure suffering tension while only the compression existed in the lower parts of SIMP structures.

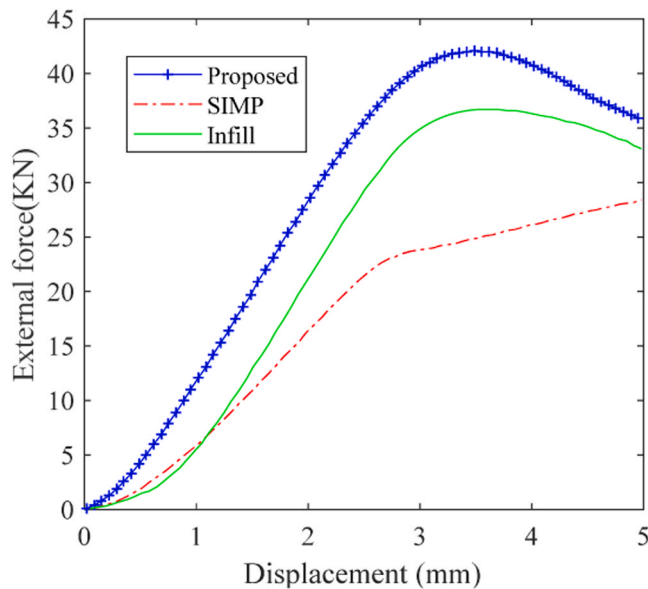
##### 3.1.2. Example 2

The second example is constructed to illustrate the performance of





**Fig. 7.** Numerical analysis and mechanical experiments on structural topology optimization with SIMP [32] (i), infill structure with buckling constraints [23] (ii) and the presented functionally-graded lattice structure (iii). The first column shows the optimal structures and the second column gives the buckling failures of the first buckling mode, the third column exhibits principal stress and the subplot (d)–(f) record the structure buckling process based on mechanics experiments.



**Fig. 8.** The external force and displacement diagram of the optimized structure by conventional SIMP, the infill method and the proposed method.

the proposed method with the configuration of T shape design area is shown in Fig. 4(a). We applied the concentrate load  $F = 0.1$  N at both of the left and right side of the bottom of the up rectangle, and the element size of discretization is  $0.01 \times 0.01$ . The Young's modulus, Poisson ratio, and the filter radius is set as stated before, and the volume fraction is set to 0.5, and the buckling load, a dimensionless parameter, is 1.1. Fig. 4(b) and (c) give the optimal results by the proposed method and its principal stress, and Fig. 4(d) and (e) display the conventional SIMP structure and the distribution of principal stress respectively. It can be observed that the functionally graded lattice patterns shown in the proposed method which consists of tensile stress that will improves the stability of the structures against buckling failures when compared with the SIMP method.

### 3.1.3. Example 3

Another illustrative example of slender shape with compression is employed for demonstrating the proposed method to construct the functionally graded lattice structure. The configurations of the design area are shown in Fig. 5(a). The concentrated load  $F = 0.1$  N is applied on the middle of the top side, and the design domain is discretized into  $20 \times 200$  elements. The buckling load factor, Young's modulus, Poisson ratio, volume fraction and the filter radius are set as 3.9, 1, 0.3, 0.3 and 0.005 respectively. Fig. 5(b) and (c) give the optimal results and the distribution of the principal stress by the proposed method and Fig. 5(d) and (e) present the conventional SIMP structures and its principal stress. It can be noticed that the proposed method construct strut patterns and tensile stress distributes in the lattice structures, which improves the structures performance against buckling failures.

### 3.2. Mechanics experiments

In order to validate the mechanics property of the optimal structure designed by the proposed method, the quasi-static compression experiments are conducted to record mechanical response at room temperature, and the measuring accuracy of equipment is 0.5%, and the loading rate is set as 2 mm/min for all the experimental parts which were printed by polylactic acid (PLA). To demonstrate the effectiveness of the proposed method, the optimal structure of conventional SIMP method and infill method are introduced for comparing the performance of structure stiffness and stability. All the experimental results were solved by MATLAB and COMSOL Multiphysics.

Fig. 6 exhibits the design area under a unit distributed load, the height and width of rectangle domain are 2 and 1 respectively, and it is fixed at the below side. The discretization is set up with  $200 \times 100$  equally-sized square elements, and Young's modulus, Poisson ratio, and the filter radius is set as stated in Section 3.1.1, the buckling constraint  $\lambda = 1.8$  and the upper bound of volume fraction is set to 0.5. The length, width and height of the manufactured parts are 50 mm, 25 mm and 100 mm respectively.

Fig. 7(a)–(f) display the numerical analysis results and mechanics experiments of SIMP structure, infill structure and functionally graded lattice structure, respectively. In Fig. 7(a), the optimized structures are presented and Fig. 7(b) exhibits the lowest buckling mode of optimized structures by numerical computation, then principal stress are displayed in subfigure(c), the tensile and compressive stresses are depicted by red and blue colors, respectively. Fig. 7(d)–(f) record the process of structure buckling failures. It can be found that the mechanics experimental results act in accordance with the numerical analysis, which mutually proved the effectiveness of the proposed method.

Fig. 8 illustrates the displacement over the external force for optimized structures with topology optimization of SIMP, infill and the presented method, respectively. It can be found that the stability of the one by the traditional SIMP method is worse than the infill method with buckling constraints and the functionally graded lattice structure of the proposed method, of which the maximum elastic limit force is 20.60 kN while the elastic limit of the corresponding external force of the infill structure and the functionally graded lattice structure are 31.50 kN and 34.60 kN, respectively. Perceptually, the SIMP result performs two individual struts which is unstable when suffers from external force and prone to buckling failure, thus its maximum load force 20.60 kN is much smaller compare to the infill method and the proposed method. Both the optimized structures of infill method and the proposed method present continuous strut patterns, of which the stability is much better than the conventional SIMP result. Furthermore, the proposed method over the infill method about 10% of the maximum load force and without introducing the local volume constraints for the construction of functionally graded lattice structure. Overall, the proposed method not only can construct the functionally graded lattice structure by using the topology optimization method via SIMP with buckling constraints, but also can improve both the stiffness and stability of lattice structures.

### 4. Discussion and future work

With illustrative examples of numerical analysis, the effectiveness of the proposed method to construct functionally graded lattice structures is demonstrated. Furthermore, the mechanics experiments are conducted to illustrate the mechanical characteristics of the functionally graded lattice structure, infill structure and the conventional SIMP structure. Both the numerical examples and mechanics experiments indicate that the proposed topology optimization model with buckling load factor constraints under the linearized buckling criteria perform better than the conventional SIMP method and the infill method. The maximum load force is about 1.68 times of the conventional SIMP method and 1.1 times of the infill method. The proposed method not only can construct lattice structures by simply extending conventional

SIMP model, but also avoid involving the local volume constraints compared to the infill method (our previous work) [35].

The proposed method for topology optimization of functionally graded lattice structure considering structure buckling can be widely used in the design of the key components for vehicle and aerospace equipment, which can greatly improve both the stiffness and the stability of the structure. In the future work, the proposed method can be extended to the design of train body which can improve the structure property of buckling resistance and energy absorber when happening crash.

### CRedit authorship contribution statement

**Long Liu:** Writing – original draft. **Bing Yi:** Conceptualization, Methodology. **Tianci Wang:** Mechanical experiments. **Zhizhong Li:** Numerical analysis. **Junhui Zhang:** Data process. **Gil Ho Yoon:** Writing – review & editing.

### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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