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# Unified Analysis with Mixed Finite Element Formulation for Acoustic-Porous-Structure Multiphysics System

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This research aims to develop a novel unified analysis method for an acoustic-porous-structure multiphysics interaction system when the porous medium is modeled by the empirical Delany–Bazley formulation. Multiphysics analysis of acoustic structure interaction is commonly performed by solving the linear elasticity and Helmholtz equations separately and enforcing a mutual coupling boundary condition. If the pressure attenuation from a porous material is additionally considered, the multiphysics analysis becomes highly intricate, because three different media (acoustic, porous, and elastic structures) with different governing equations and interaction boundary conditions should be properly formulated. To overcome this difficulty, this paper proposes the application of a novel mixed formulation to consider the mutual coupling effects among the acoustic, fibrous (porous), and elastic structure media. By combining the mixed finite element formulation with the Delany–Bazley formulation, a multiphysics simulation of sound propagation considering the coupling effects among the three media can be easily conducted. To show the validity of the present unified approach, several benchmark problems are considered.

*Keywords*: Acoustic-porous-structure interaction; Delany–Bazley model; acoustic analysis; empirical material model.



## 1. Introduction

To reduce the adverse effect of loud noise or to improve static and dynamic characteristics of a broad range of engineering structures,[1–6] a number of finite element analysis and design methods have been developed and proposed with the help of ever-increasing availability of high-speed computers.[1,6–9] Despite a number of studies relevant to the analysis methods for acoustic, acoustic-structure, and acoustic-porous-structure based on Biot's theory,[10–13] to the best of our knowledge, a unified analysis for an acoustic-porous-structure interaction system with *empirical* material model for fibrous medium has not been considered. This paper presents a new analysis method based on a mixed finite element formulation to consider acoustic-porous-structure interaction using the Delanzy–Bazley material model, which is a representative empirical material model for acoustic pressure attenuation inside a fibrous medium.[4,5,9,14,15] With the unified mixed finite element analysis approach, it is possible to conduct a unified analysis for improving pressure attenuation by simultaneously distributing porous and linear elastic media. This aspect is very important in structural optimization, especially topology optimization, which allows the creation or deletion of structural, acoustic, and porous domains (Fig. 1).[16]

It is important to control and reduce the adverse effects of loud and irritating noises that lead to adverse health issues in industry and daily life (Fig. 2). For instance, a factory with loud machinery noise often causes ear health problems for its workers.[6,14,17–19] To reduce the adverse effects of loud and irritating noises, many engineering approaches have been developed (Figs. 1 and 2). For example, some acoustic structures (e.g. expansion chambers, Helmholtz's resonators, fibrous textured wall surfaces) have been used to increase acoustic pressure attenuation and shield some areas from incoming sound waves in machine and

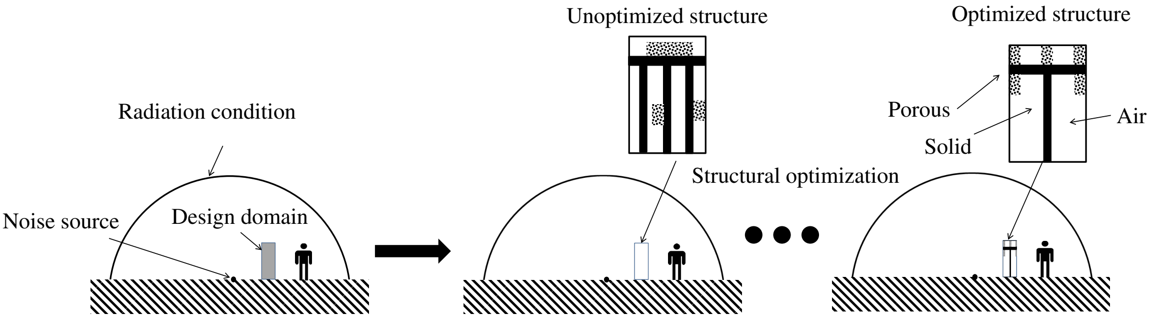



Fig. 1. Analysis domain changes in structural optimization.

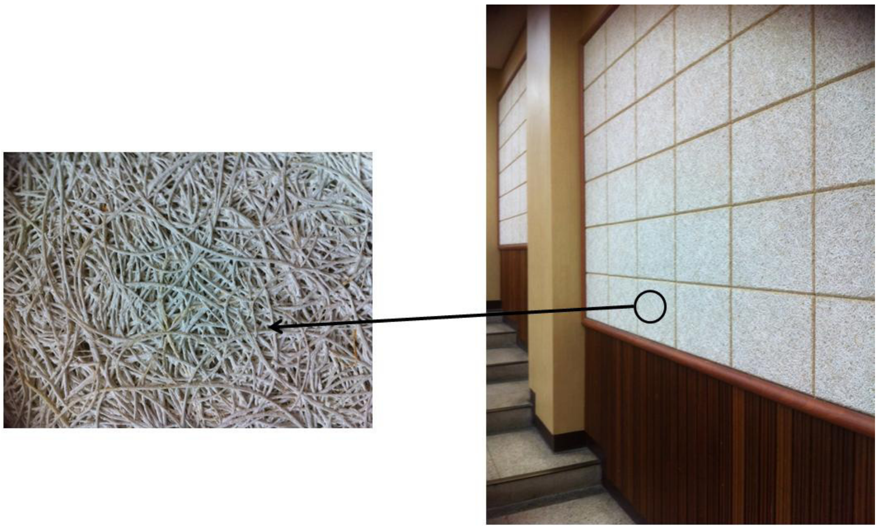


Fig. 2. Absorbing material in a lecture room at Hanyang University.

architecture applications.[6] Often small vibrating structures whose resonance frequencies match those of problematic noises and vibrations are implemented to absorb structural vibrational energy and consequently reduce their propagation. To identify noise phenomena clearly and to resolve them by means of acoustic engineering approaches, the basic physical principles, such as the dimensions of spaces, wavelength, frequency, and sound wave coupling between elastic structures and fibrous media should be considered. Particularly, when it is possible to neglect the couplings between acoustic domains and the structures enveloping acoustic domains, the Helmholtz equation can be used to numerically estimate noise levels for some objective regions of interest.

When the velocity of a fluid plays an important role in the acoustic phenomena, direct analysis using the Navier–Stokes equations coupled or uncoupled with the Helmholtz equation can be employed (see Ref. 6 and references therein). However, one interesting acoustic phenomenon is the coupling effect between the elastic structures and the fibrous media when the elastic structure is not considered to be rigid with infinite impedance. For vibro-acoustic systems, the mutual couplings between the acoustic and structure or among the acoustic, structure, and fibrous media should be considered. One popular coupling theorem for this multiphysics system is Biot's theory, which models micro-scale interactions between the structure and a fluid.[6,13]

Previous researches indicated that acoustic–structure interaction simulation can be conducted in the framework of a mixed finite element formulation.[1–4] Furthermore, this method has been used for the simulations of incompressible or nearly incompressible elastic media and vibro-acoustic interaction problems.[1–4] In the framework of this mixed formulation, both pressure and displacements become the primary variables with the linearized Euler's and equilibrium equations. Therefore, it was shown that by adopting a single domain with heterogeneous material properties in the framework of the mixed formulation, the acoustic–structure interaction phenomena can be simulated.[1–4,20] In this research, the mutual couplings among the fibrous, acoustic, and structure materials are additionally considered by parameterizing the material properties of the mixed formulation (Fig. 1). To consider pressure attenuation from fibrous materials, the Delany–Bazley material model is formulated in the framework of the present mixed finite element formulation, which is one of the main contributions of this paper.

In the unified mixed formulation, this study straightforwardly considered three interaction phenomena among the fiber, acoustic, and structure materials. First, the acoustic–structure interaction phenomena are satisfied in the mixed formulation.[1–4,20] Second, the coupling conditions between the acoustic and the fibrous media are satisfied by adopting a single domain with heterogeneous material properties (bulk modulus, shear modulus, and density) according to the domain definition of the Helmholtz equation, which is approximately simulated by the mixed formulation (Fig. 1). The Delany–Bazley material model, which is applicable to fibrous media with porosities close to one, was used.[4,5,9,14,15] This empirical material formulation has been widely used and has shown its validity in many acoustic engineering applications.[5,10,11,13] A benefit of the Delany–Bazley model is that the pure Helmholtz equation can be used for the computational calculation of pressure propagation and pressure attenuation due to a fibrous material.[4,5,9,14,15] Finally, the coupling conditions between the fibrous material and the structure media are satisfied, because the fibrous material is treated as a special case of an acoustic domain with complex density and bulk modulus. By adopting three heterogeneous material properties (i.e. density, bulk modulus, and shear modulus) in the mixed formulation, this research found that it is possible to simulate the mutual couplings among the porous, acoustic, and elastic structures when the material properties of the elastic structure, air, and porous materials are assigned to the unified mixed finite element formulation rather than integrating three different equations (Fig. 1).

This research is organized as follows: first, the basic formulations of acoustic structure interaction are provided, and then the porous material model (the Delany–Bazley model) is introduced. Next, the present mixed formulation is presented for the multiphysics system of fibrous, acoustic, and structure media, and several acoustic analysis examples are then presented. Finally, the findings and future research topics are summarized.

## 2. Governing Equations for Acoustic–Fibrous–Structure Interaction

### 2.1. *Basic governing equations*

Acoustic simulation with Helmholtz equation for homogeneous acoustic domain (nonfibrous media).

Acoustic pressure propagation in a homogeneous acoustic medium (neglecting the dissipation of acoustic energy) is normally simulated by solving the following Helmholtz equation under proper boundary conditions[6] and assuming harmonically varying pressure, i.e. $\tilde{p}(t) = pe^{i\omega t}$.

$$\nabla \cdot \left(\frac{1}{\rho_a}\nabla p\right) + \frac{\omega^2 p}{\rho_a c_a^2} = 0 \quad \text{on } \Omega_a, \tag{1}$$

where $p$, $\rho_a$, and $c_a$ are the pressure in the acoustic domain $\Omega_a$, density of the acoustic domain, and local speed of sound, respectively. The angular velocity and wave number are denoted by $\omega$ and $k(k = \frac{\omega}{c_a})$, respectively. With an appropriate numerical solution procedure such as the finite element method or finite volume method, the acoustic pressure propagation phenomenon can be simulated for acoustic engineering problems. Although the finite element method can solve the Helmholtz equation for a complex and arbitrarily shaped system, the Galerkin finite element analysis with lower-order basis often suffers from numerical error and dispersion error caused by the inaccurate prediction of pressure inside finite elements for medium and high wave numbers, which is one of the restrictions of the finite element method.[19] To improve the accuracy and stability, the number of elements and the subsequent computational efforts should be increased. To overcome this difficulty, some advanced methods such as the Trefftz method for the Helmholtz equation have been reported. (See Ref. 19 and the references therein.)

If there is radiation or scattering from an elastic structure toward the surrounding fluid, the mutual coupling between the elastic body and the surrounding fluid should be taken into account. (See Refs. 6 and 7 for general pressure, acceleration, and impedance boundary conditions.) At the interaction boundary, the local balance of the linear momentum equation should be satisfied as follows:

Interface condition for the acoustic domain:

$$\mathbf{n} \cdot \nabla p = \omega^2 \rho_a \mathbf{n}^{\mathrm{T}} \mathbf{u} \quad \text{in } S_{\mathrm{int}}, \tag{2}$$

where $S_{\mathrm{int}}$ is the interfacing boundary, and $\mathbf{n}$ is the outgoing normal vector. The structural displacement at the interaction boundary is denoted by $\mathbf{u}$.

### 2.1.1. *Governing equation*: *Linear elasticity problems*

Time-harmonic linear structural analysis neglecting the body force can be described by Newton's law:

$$\nabla \cdot \boldsymbol{\sigma} = -\omega^2 \rho_s \mathbf{u} \quad \text{on } \Omega_s, \tag{3}$$

where $\boldsymbol{\sigma}$, $\rho_s$, and $\mathbf{u}$ are the stress tensor in the structural domain $\Omega_s$, structural mass density, and displacement vector, respectively. At the interface of the structural domain, the traction of the solid part should equal the pressure, and the following condition should be imposed.

Interface condition for the solid part:

$$\mathbf{f}_{S_{\mathrm{int}}} = p\mathbf{n} \quad \text{on } S_{\mathrm{int}}. \tag{4}$$

The structural force applied at the interaction boundary is denoted by $\mathbf{f}_{S_{\mathrm{int}}}$. After imposing the interface boundary conditions of Eqs. (2) and (4), the scattering wave and structural response can be calculated by a standard finite element procedure. Figure 3 shows the overall procedure of the common acoustic-structure interaction analysis under the interaction boundary conditions.

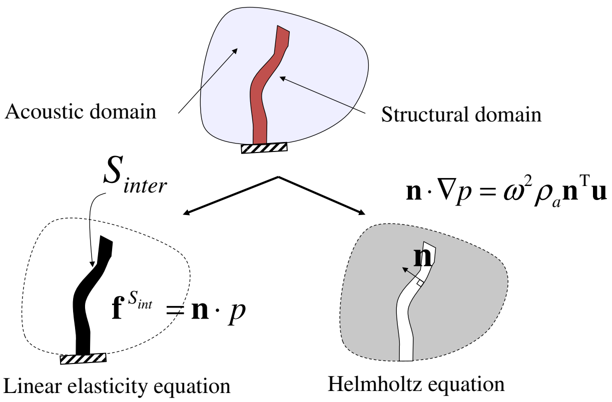



Fig. 3. Governing equations and interaction boundary conditions between acoustic and structural domains.

### 2.1.2. *Acoustic simulation with empirical material model for fibrous media*

The classical Helmholtz Eq. (1) is derived from the linearized Euler's equation for compressible media by neglecting the dissipation of acoustic energy. However, often the acoustic energy dissipation from absorption and attenuation of sound pressure by a fibrous medium need to be considered in some practical acoustic engineering simulations,[21] and many theoretical and numerical models have been developed to simulate the dissipative energy loss from various fibrous materials.[4,5,9,12,14,15] These models can be categorized into several classes. One class for such fibrous material simulation is the *phenomenological* approach, which directly simulates the viscous and thermal interactions between air and fibrous materials. Biot's theory is a popular phenomenological approach.[13,21] Despite some theoretical advantages, this approach requires the values of several parameters determined by the geometry and material properties of the fibrous material of interest. Another shortcoming from a computational point of view is that it requires more degrees of freedom (DOF) than the Helmholtz equation.[7] In addition, the geometric parameters of a fibrous material are often too irregular to be measured and used in practice.

According to Bolton,[9,14] most fibrous materials are inhomogeneous, and their material properties vary randomly throughout their volume. Another class for the simulation of a fibrous material may be the empirical material formulation based on the Helmholtz equation with complex material properties.[10,11,22] This research considers the Delany–Bazley material formulation, which is known as one of the most representative empirical material formulations for fibrous materials with porosities close to one, and is formulated as follows:

$$k_c = k_a(1 + 0.0978(\tilde{f})^{-0.7} - i0.189(\tilde{f})^{-0.595}), \tag{5}$$

$$Z_c = Z_a(1 + 0.057(\tilde{f})^{-0.734} - i0.087(\tilde{f})^{-0.732}), \tag{6}$$

$$\tilde{f} = \frac{\rho_a f}{\xi} c_c = \frac{\omega}{k_c}, \quad \rho_c = \frac{k_c Z_c}{\omega}, \quad f = \frac{\omega}{2\pi}, \tag{7}$$

where $k_a$ and $\rho_a$ denote the wave number and the density of air without pressure attenuation, respectively. The complex wave number and impedance value of the Delany–Bazley empirical material model are $k_c$ and $Z_c$, respectively. Note that the empirical material formulas shown in Eqs. (5)–(7) are based only on the measurements of the bulk airflow resistivity, $\xi$, which is highly dependent on the chosen fibrous material and introduces viscous losses in sound propagation.[11,12] Because of its simplicity in numerical implementation, this Delany–Bazley empirical material model is widely accepted and works well for fibrous materials when the normalized frequency ($\tilde{f} = \frac{\rho_a f}{\xi}$) is in the range of $[0.01, 1]$. Nevertheless, the above empirical formulation does not guarantee a suitable prediction of the acoustic behavior of all porous materials in all frequency ranges, particularly for $\tilde{f} > 1$ or $< 0.01$.[11,15] Furthermore, the coefficients proposed by Delany and Bazley have been adjusted to apply the formula to other poro-elastic materials. For example, Miki[21] and Allard and Champoux[20] obtained different coefficient values and formulas that improve the prediction accuracy of the Delany and Bazley model by incorporating material characteristics and other geometric quantities such as tortuosity and the shapes of micro-holes inside fibrous materials.[15,21]

### 2.2. *Unified mixed finite element formulation for acoustic-porous-structure interaction problems*

The mixed finite element procedure was extended to simulate the acoustic-porous-structure interaction. To the best of the author's knowledge, this approach has not been tried prior to this research. In the mixed finite element formulation, the governing equations without body forces are formulated as follows:

$$\nabla \cdot \boldsymbol{\sigma} = -\omega^2 \rho \mathbf{u} \quad \text{on } \Omega, \tag{8}$$

$$p = -K\varepsilon_v, \tag{9}$$

$$\mathbf{e} = \boldsymbol{\varepsilon} - \frac{\varepsilon_V}{3}\boldsymbol{\delta}, \tag{10}$$

$$\varepsilon_v = \frac{\Delta V}{V} = \varepsilon_{kk}, \tag{11}$$

$$K = \frac{E}{3(1-2\nu)}, \quad G = \frac{E}{2(1+\nu)}, \tag{12}$$

$$\boldsymbol{\sigma} = \begin{cases} -p\boldsymbol{\delta} + 2G\mathbf{e} & \text{for incompressible material} \\ K\varepsilon_v\boldsymbol{\delta} + 2G\mathbf{e} & \text{for compressible material} \end{cases}, \tag{13}$$

where $K$, $G$ and $\rho$ are the bulk modulus, shear modulus, and mass density, respectively, and $\boldsymbol{\delta}$ is Kronecker's delta and the strain tensor is denoted by $\boldsymbol{\varepsilon}$ in Eq. (11). The above finite element equations have been used for incompressible media, and by varying the shear modulus $G$ and bulk modulus $K$, the acoustic and structural domains can be described for the multiphysics simulation of both $G$ and $K$ simultaneously.[1–4,20] To illustrate this aspect, a two-dimensional analysis was considered. By assigning zero for the shear modulus, it is possible to simulate the incompressibility condition of the acoustic domain.

For the finite element implementation, the weak forms are used.

$$\int_\Omega \delta\boldsymbol{\varepsilon}^T \boldsymbol{\sigma} d\Omega + \int_\Omega \omega^2 \rho \delta\mathbf{u}^T \mathbf{u} d\Omega = 0 \tag{14}$$

$$\int_\Omega \left(\frac{p}{K + \varepsilon_V}\right) \delta p d\Omega = 0 \tag{15}$$

where the virtual displacement, the virtual deviatoric strain, and the virtual strain are denoted by $\delta\mathbf{u}$, $\delta\varepsilon_V$, and $\delta\boldsymbol{\varepsilon}$, respectively. To satisfy the so-called Inf-sup condition, the continuous displacements and the continuous pressure element are implemented as shown in Fig. 4.[1–3] With this, the standard finite element formulation can be obtained.

$$\begin{bmatrix} \mathbf{K}_{uu} - \omega^2 \mathbf{M}_{uu} & \mathbf{K}_{up} \\ \mathbf{K}_{up}^T & \mathbf{K}_{pp} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{P} \end{bmatrix} = \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix} \tag{16}$$

where the structural displacements and the pressure vectors are denoted by $\mathbf{U}$ and $\mathbf{P}$, respectively. The stiffness matrices for the displacements and the pressure, the coupling

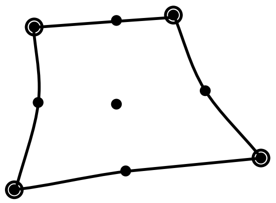



Fig. 4. Mixed finite element with continuous displacements and continuous pressure.

matrix, the mass matrix, and the external force vector are denoted by $\mathbf{K}_{uu}$, $\mathbf{K}_{pp}$, $\mathbf{K}_{up}$, $\mathbf{M}_{uu}$, and $\mathbf{R}$, respectively.

The analysis domain $\Omega$ is decomposed as:

$$\Omega = \Omega_s \cup \Omega_a \cup \Omega_p, \tag{17}$$

where the subscripts $s$, $a$ and $p$ denote structural, acoustic, and fibrous, respectively. The following material properties of the mixed formulation was assigned.[1–4,20]

$$K \equiv K_s, \quad G \equiv G_s, \quad \rho \equiv \rho_s \quad \text{on } \Omega_s, \tag{18}$$

$$K \equiv K_a, \quad G \equiv G_a = 0, \quad \rho \equiv \rho_a \quad \text{on } \Omega_a, \tag{19}$$

$$K \equiv K_p, \quad G \equiv G_p = 0, \quad \rho \equiv \rho_p \quad \text{on } \Omega_p, \tag{20}$$

In the fibrous domain, the shear modulus becomes zero, and the bulk modulus and density become complex values, as given by the Delany–Bazley material model. The present mixed formulation provides a means for rapid and easy consideration of the multiphysics effects of changes in the shape and topology of the acoustic domain (Figs. 3 and 5). To meet the so-called inf–sup condition, the continuous Q2 displacement and the continuous Q1 pressure element are implemented (Fig. 4).[1–3] Note that it is possible to use different finite element spaces for $\mathbf{u}$ and $p$ for the structural domain. In other words, different shape functions can be used for the solid, porous, and acoustic materials. For example, one study has combined the standard finite element method with the wave-based method (an indirect Trefftz method).[19] In the present simulation, we employed the same finite element spaces for all materials.

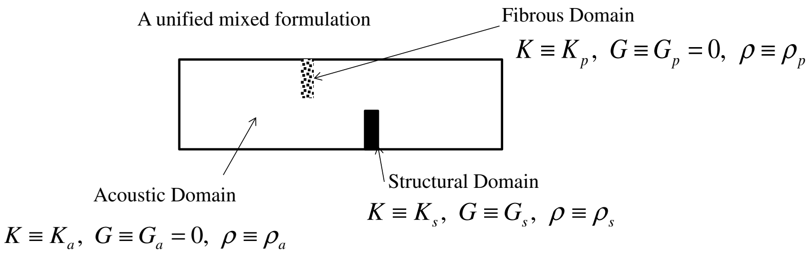



Fig. 5. Material assignment for acoustic-porous-structure simulation.

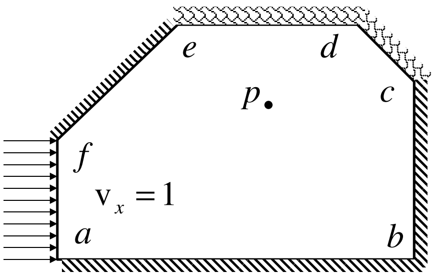
## 3. Multiphysics Analysis Examples

To show the validity of the present mixed formulation for the coupled analysis of acoustic, fibrous, and structure media, the following benchmark acoustic analysis examples are considered. [See Refs. 1, 3, 4 and 7 for the application of the mixed formulation for acoustic (fluid)-structure.]

### 3.1. *Acoustic pressure calculation of simplified car*

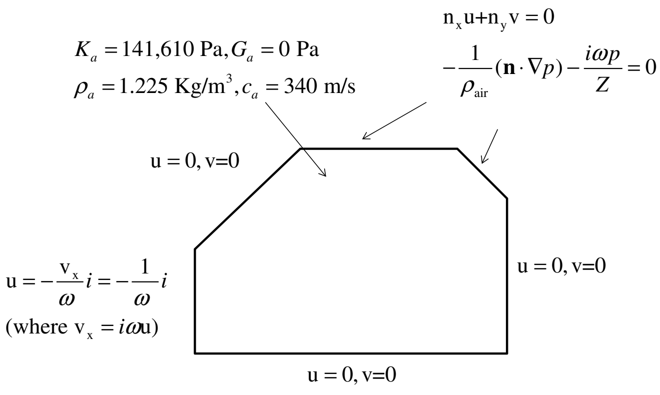
#### *Car model* 1

For the first numerical example, the pressure values of a simplified car cavity model are calculated (Fig. 6); the acoustic domain is filled with air and bounded in part by velocity input, a rigid wall, and elsewhere by a sound-absorbing wall with an acoustic impedance $Z$.[19,23] Although this is only a simplified pure acoustic car model, this example was chosen because it shows the various aspects of the present formulation. For a noise source, the velocity $v_{in} = 1\,\mathrm{m/s}$ was assigned to the left panel at $x = 0\,\mathrm{m}$. At the roof, the




| point | $x$ | $y$ |
|---|---|---|
| $a$ | 0.00 | 0.00 |
| $b$ | 1.50 | 0.00 |
| $c$ | 1.50 | 0.75 |
| $d$ | 1.25 | 1.00 |
| $e$ | 0.50 | 1.00 |
| $f$ | 0.00 | 0.50 |
| $p$ | 0.88 | 0.70 |

(a)




(b)

Fig. 6. Two-dimensional simplified car cavity analysis example.[19,23] Here, $\rho_a = 1.225\,\mathrm{Kg/m^3}$, $c_a = 340\,\mathrm{m/s}$, the number of DOF of mixed formulation = 162 767, and the number of DOF of the Helmholtz equation (the classical Galerkin method for the Helmholtz equation) = 50 169.

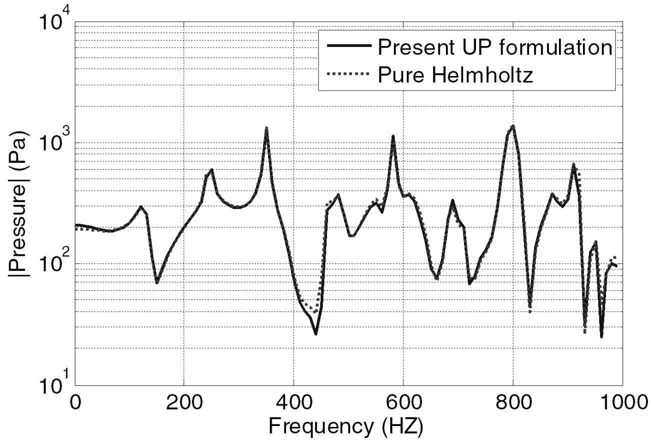



Fig. 7. The comparison of the pressures.

normal impedance $Z = \rho_a c_a$ was introduced for the attenuation effect by air. Figure 7 shows the obtained pressure values at point $P$ that are similar to the results in Ref. 23. Similar responses can be obtained with the Helmholtz equation and the present mixed formulation.

In Fig. 8, the solution convergence with respect to mesh refinements is tested. As the additional primary variables are added, the number of DOF is much greater than that

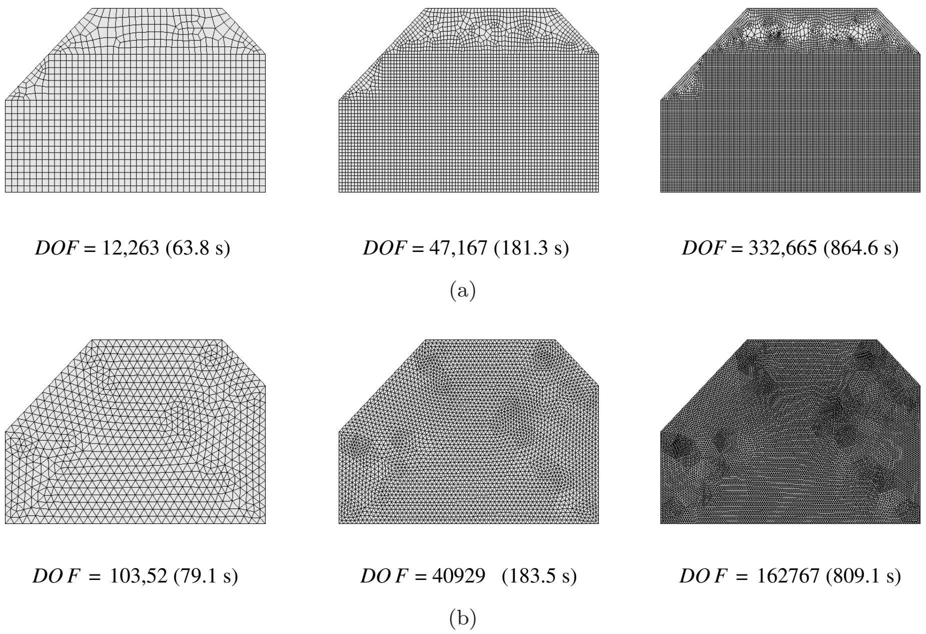
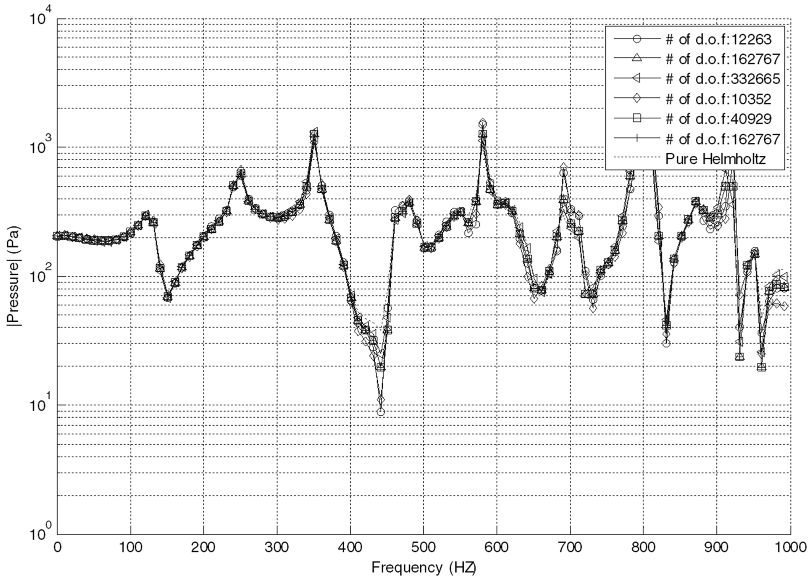



Fig. 8. Mesh refinement results: (a) quadrilateral finite element meshes, (b) triangular finite element meshes and (c) solutions.




(c)

Fig. 8. (*Continued*)



of the pure Helmholtz equation. From a computational point of view, the present mixed formulation takes much more time than the pure Helmholtz equation. In this problem, it takes 63.8, 181.3 and 864.6 s for the FE models with 12 263, 47 167 and 332 665 DOF, respectively. The triangular meshes take 79.1, 183.5 and 809.1 s with 10 352, 40 929 and 162 767 DOF, respectively.

***Car model* 2**

For the next numerical example, the simplified car in Fig. 9 is considered here; the geometry is redrawn by the open software (XY scan) and the eigenfrequencies of the simplified car are similar to those of the reference.[18] The acoustic domain is again filled with air and bounded in part by the velocity input ($v_{in} = 1\,\mathrm{m/s}$) at the left side, by the sound-absorbing wall with the acoustic impedance $Z = \rho_a c_a$ at the top roof and elsewhere by the rigid wall. For the comparisons, the pressure inputs at the point A and the point B inside the car cavity are compared by the Helmholtz equation and the present mixed formulation in Figs. 9 and 10. The finite element mesh and the boundary conditions are presented in Fig. 10. As in the first simplified car, the almost similar responses of the Helmholtz equation and the mixed formulation can be obtained.

***Car model* 3**

For the last car example, the other simplified car in Fig. 11(a) is considered here; the geometry is also redrawn by the open software (XY scan).[24] The compartment has the rigid wall boundary condition for all the boundaries except the left top surface for the noise source from engine room as shown in Fig. 11(a). To calculate the pressure inside the car

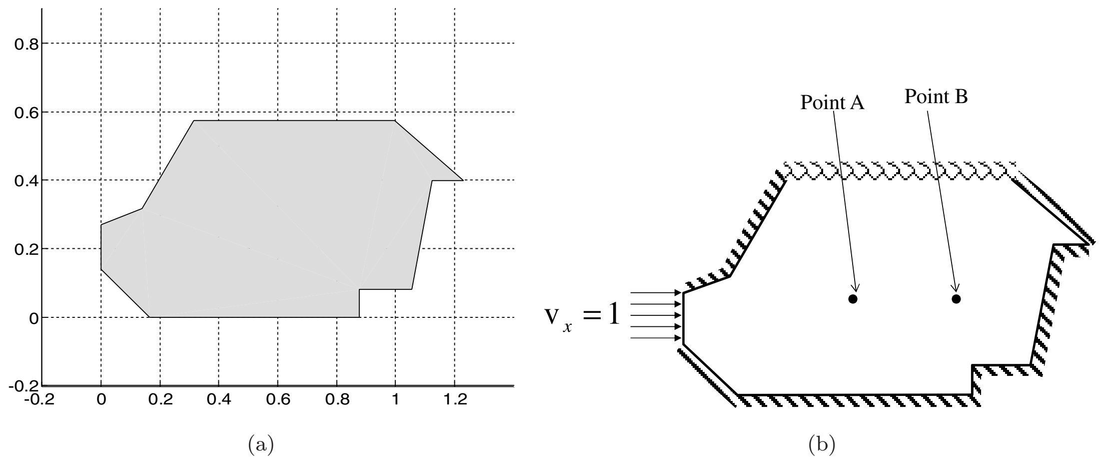



Fig. 9. The 2nd two-dimensional simplified car cavity analysis example.[18] (Scanned geometry from Ref. 18 by the digitizer program XY scan, $\rho_a = 1.25\,\mathrm{Kg/m^3}$, $c_a = 340\,\mathrm{m/s}$, computed eigenfrequencies = [0 Hz, 174.89 Hz, 306.32 Hz, 321.5 Hz, 419.34 Hz and 450.61 Hz]).

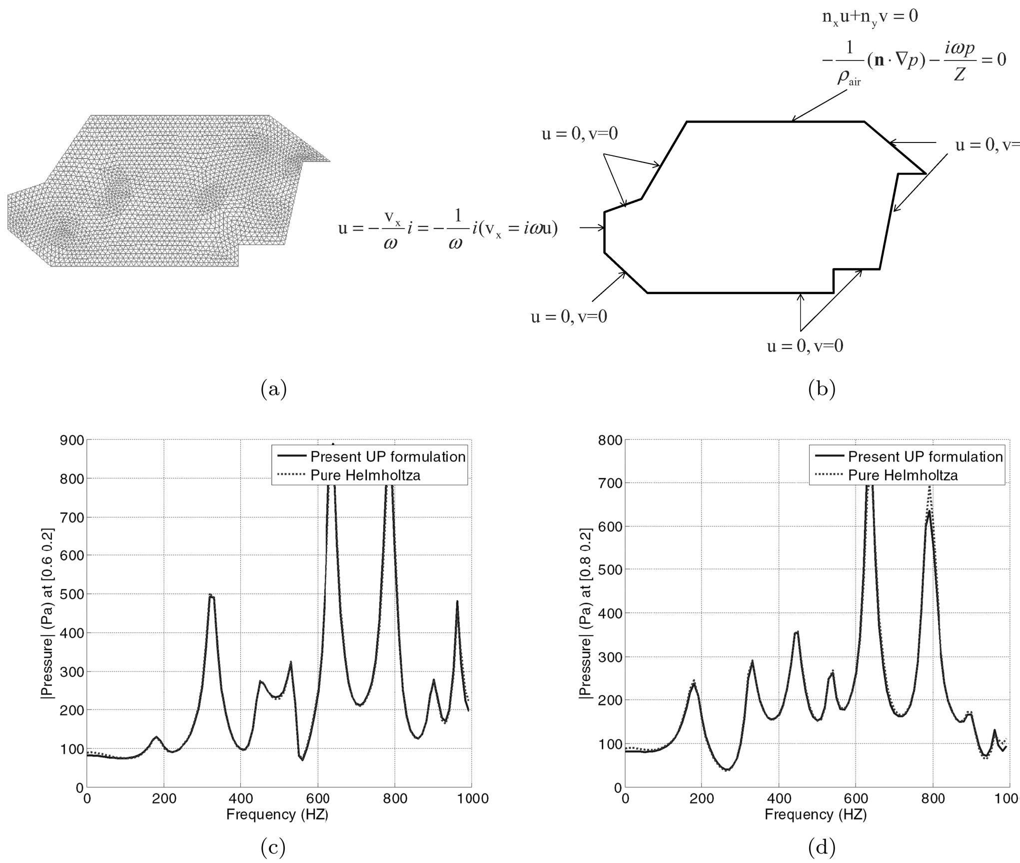



Fig. 10. The analysis of the simplified car of Fig. 9. (a) a finite element mesh, (b) the boundary condition, (c) and (d) the responses of the point A (0.6, 0.2) and the point B (0.8, 0.2).

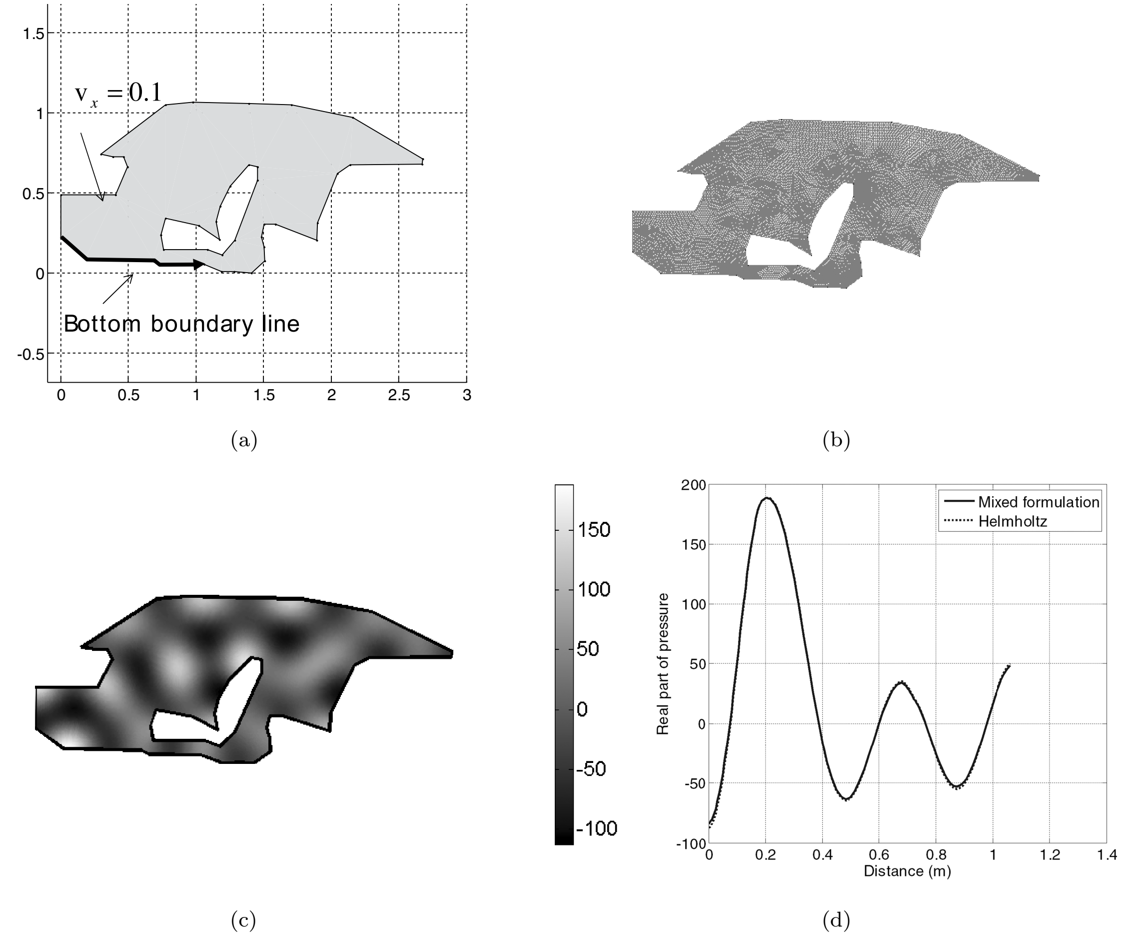



Fig. 11. The 3rd two-dimensional simplified car cavity analysis example.[24] (Scanned geometry from Ref. 24 by the digitizer program XY scan, $\rho_a = 1.225\,\mathrm{Kg/m^3}$, $c_a = 343\,\mathrm{m/s}$) (a) the geometry, (b) a finite element mesh and (c) the pressure distribution and (d) the solution comparison between the mixed formulation and the Helmholtz equation at the bottom boundary.

with the 15 wave number, the finite element in Fig. 11(b) is used for both the present mixed formulation and the Helmholtz equation. The pressure distribution is obtained as Fig. 11(c). Figure 11(d) shows the pressure distributions at the bottom boundary line of the mixed formulation and the Helmholtz equation. As illustrated, the almost similar distributions can be obtained.

### 3.2. *Impedance at normal incidence of layer backed by impervious rigid wall*

In the next numerical experiment, the complex impedance at the surface of a layer of fibrous material with thickness of 0.1 m and normal flow resistivity of $10{,}000\,\mathrm{Nm^{-4}s}$ in Fig. 12 is calculated.[10,25] The analytical impedance at the surface of the layer fixed on a rigid wall in

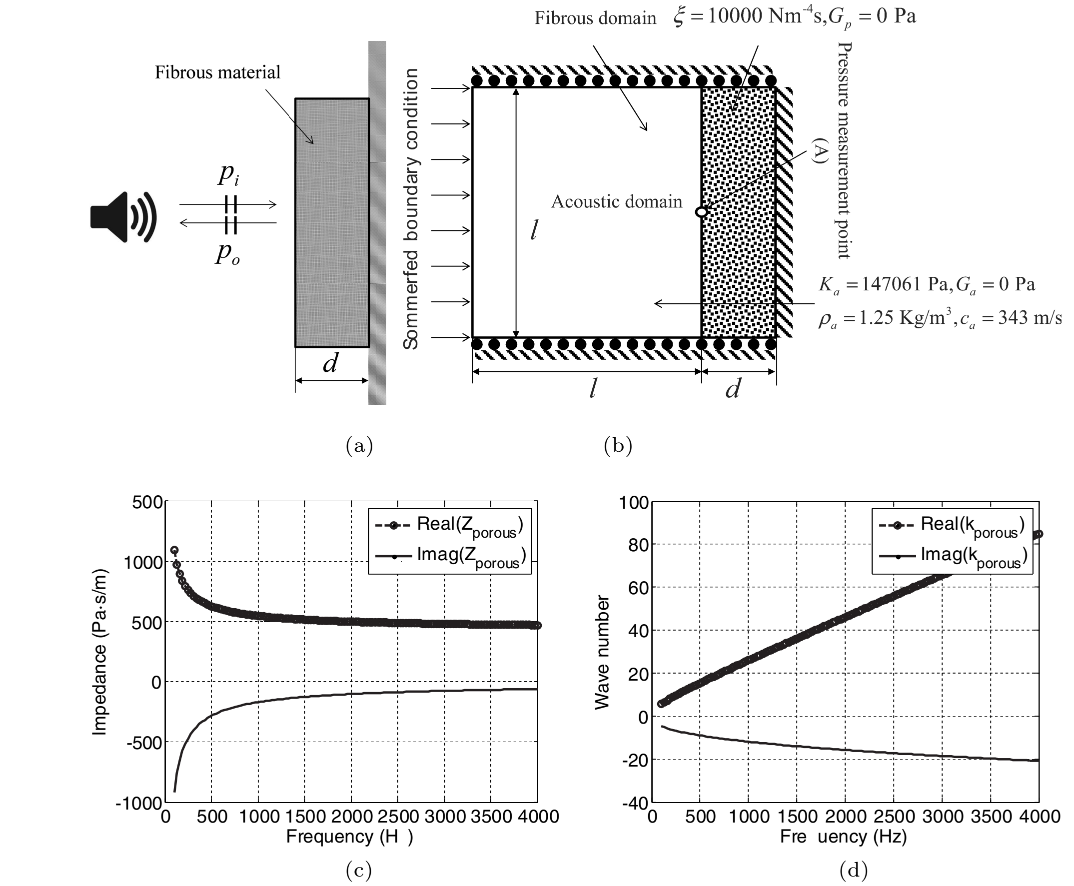
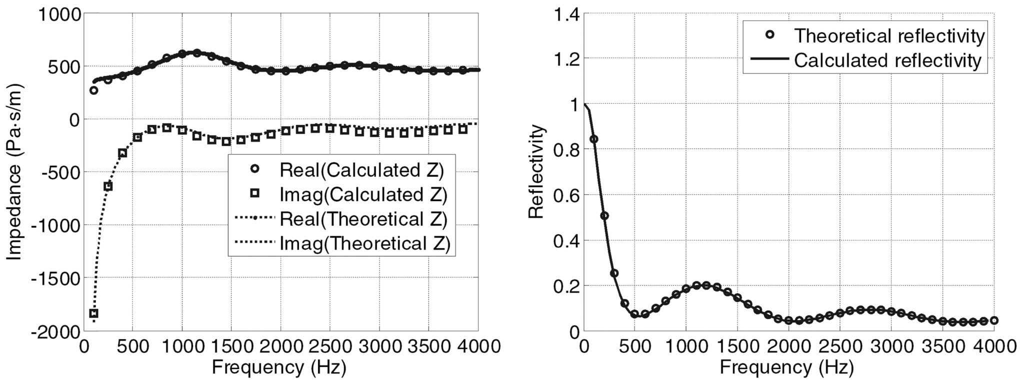



Fig. 12. Pressure attenuation by a layer of a fibrous material of thickness $d$ with normal flow resistivity of $10\,000\,\mathrm{Nm}^{-4}\mathrm{s}$ (see p. 25, Ref. 10): (a) a schematic layout to measure the absorptivity and impedance; (b) employed finite element model and Sommerfeld boundary condition for sound source; and (c) impedance and (d) wave number of porous material.

Fig. 13(a) can be derived as follows:

$$Z = -iZ_c \cot(k_c d), \tag{21}$$

where the complex impedance ($Z_c$) and complex wavenumber ($k_c$) are determined by Eqs. (5) and (6) with the flow-resistivity value $\xi = 10\,000\,\mathrm{Nm}^{-4}\mathrm{s}$, and thickness $d = 0.1\,\mathrm{m}$. Although the above simple equation is derived by the one-dimensional acoustic model assumption, a two-dimensional finite element model is constructed in Fig. 12 with the present mixed FE formulation to calculate the impedance at the surface of the layer of the fibrous material.[10,25] In the left acoustic domain, the bulk modulus and density of the air are assigned zero shear modulus in order that the model is reduced to the Helmholtz equation. In the right fibrous layer domain, complex bulk modulus and the complex density of the fibrous material are assigned. With the present monolithic mixed FE formulation, the coupling boundary conditions between the acoustic and fibrous domains are satisfied




(a) (b)

Fig. 13. (a) Complex impedance $Z$ calculated by Eq. (19) and the present approach; (b) normal incidence reflectivity from uniform fibrous layer.



in the unified FE formulation. To simulate the sound source, the normal incident pressure input is applied by the Sommerfeld boundary condition imposed at the leftmost side as:

$$\mathbf{n} \cdot \nabla p + i \cdot k \cdot p = 2i \cdot k \cdot p_{in}, \tag{22}$$

where $\mathbf{n}$ and $p_{in}$ are the outward unit vector normal to the acoustic domain and the pressure amplitude of the incoming wave, respectively, in Fig. 12(b). The impedance is defined as the ratio of the pressure to the velocity; thus, the pressure values at the mid-top surface of the layer of the fibrous material, indicated by A in Fig. 12(b), are calculated by varying the exciting frequency values, and the following simple impedance calculations are performed at point A in Fig. 13(a).

$$Z = \frac{p}{\frac{du}{dt}} = \frac{p}{i\omega u}, \tag{23}$$

where the $x$ displacement value is denoted by $u$, which is one of the components of $\boldsymbol{u}$. Therefore, the time derivative of the harmonic varying $x$ displacement (i.e. $x$ velocity) becomes $i\omega u$. Figure 13(a) shows the curves of the analytical and calculated impedance values. As illustrated, the present mixed FE formulation can predict the one-dimensional analytic impedance very accurately.

Furthermore the reflectivity value $R$ is calculated from its definition as follows:

$$R = \frac{p_o - p_i}{p_i} = \frac{Z - Z_a}{Z + Z_a} = 1 - |A|^2, \tag{24}$$

$$p_i = p_{in} e^{-ik_a x}, \tag{25}$$

where the pressure at $x = L$ and its magnitude are $p_i$ and $p_{in}$, respectively, and $A$ is the absorptivity. The acoustic impedance and wavenumber of air are $Z_a = \rho_a c_a$ and $k_a$, respectively. Figure 13(b) shows the analytical and calculated reflectivity values. As shown, the present monolithic mixed FE formulation can simulate the absorptive phenomena of the fibrous layer very accurately.

This example shows the following characteristics. At first, the multiphysics simulation of air and fibrous media can be performed in a unified mixed formulation without separating air and solid material by using different governing equations under the mutual coupling boundary condition. Although the mixed formulation requires more elements and DOF compared with the pure Helmholtz equation, it can still be useful to simulate the acoustic-porous-structure interaction system adopting a unified domain with heterogeneous material properties.

### 3.3. *Impedance at normal incidence of a fibrous layer with an air gap between the fibrous layer and the rigid wall*

In this analysis example, the complex impedance at the surface of a layer of fibrous material with thickness of 0.1 m and backed by an air gap of 0.1 m is calculated.[10] The normal flow resistivity is again set to 10 000 $\mathrm{Nm^{-4}s}$. From the one-dimensional acoustic theory, the analytical impedance at the surface of the layer in Fig. 14(a) can be derived as:

$$Z(M_2) = Z_{\text{porous}} \frac{-iZ(M_1)\cot(k_c d) + Z_{\text{porous}}}{Z(M_1) - iZ_{\text{porous}}\cot(k_c d)}, \tag{26}$$

where $k_c$ is the wave number. The complex impedance values at the positions $M_1$ and $M_2$ are denoted by Z($M_1$) and Z($M_2$), respectively. To simulate the above analytical solution, a two-dimensional finite element model is constructed in Fig. 14 with the present mixed FE formulation. Figure 15 shows the impedance and the wave number of the fibrous material.

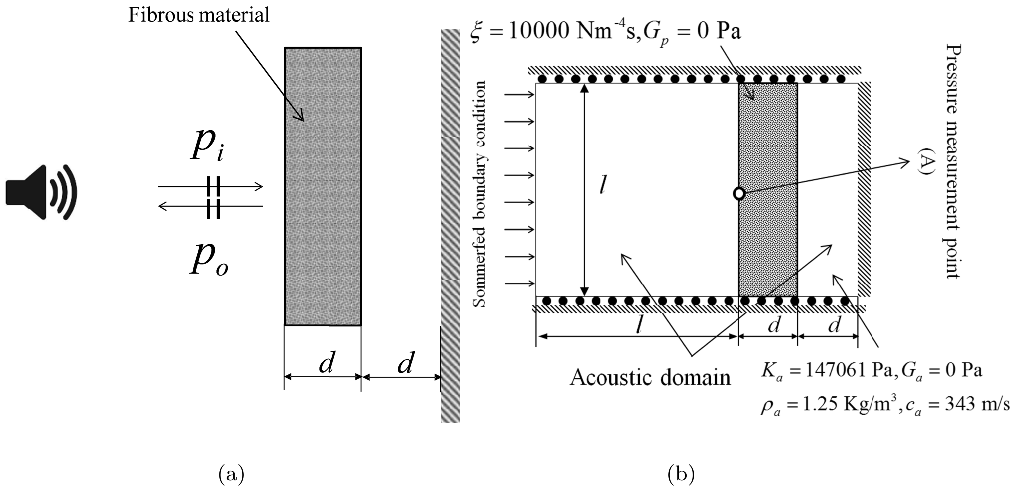
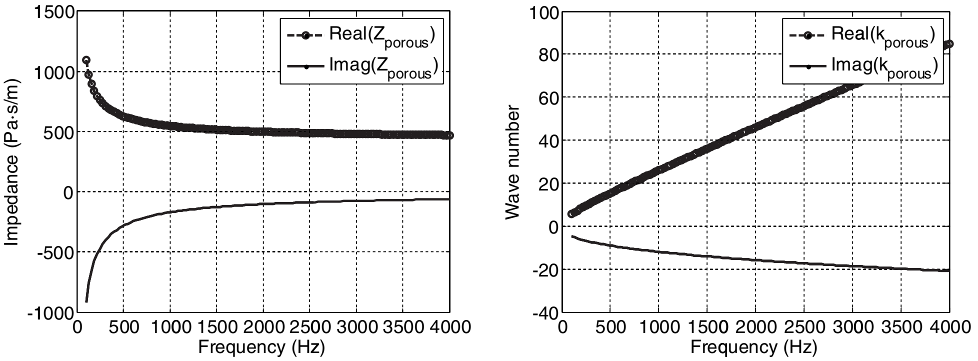
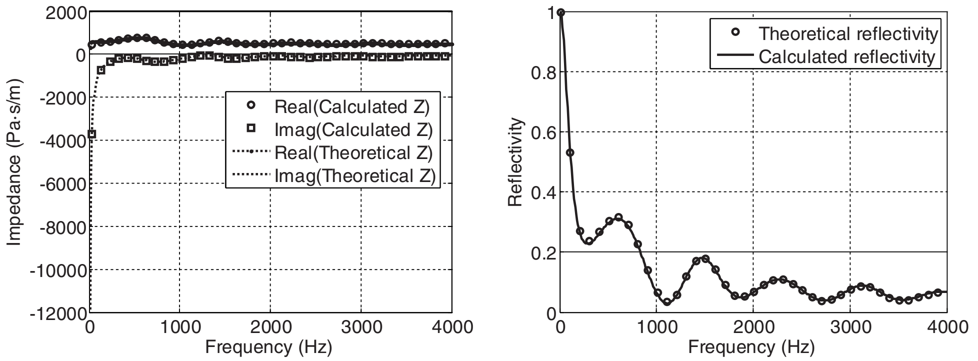



Fig. 14. Pressure attenuation of a layer of a fibrous material of thickness $d$ with normal flow resistivity of 10 000 $\mathrm{Nm^{-4}s}$ (see p. 27, Ref. 10): (a) schematic layout to measure the absorptivity and impedance; and (b) employed finite element model and Sommerfeld boundary condition for sound source.




(a) (b)

Fig. 15. (a) Impedance; and (b) wave number of porous material.




(a) (b)

Fig. 16. (a) Complex impedance $Z$ calculated by Eq. (19) and the present approach; (b) normal-incidence reflectivity from uniform fibrous layer.



As in the previous example, the impedance and reflectivity value are calculated as shown in Eqs. (23)–(25).

The mixed formulation is also used to simulate the above theoretical formulation in Fig. 14(b). The calculated impedance and reflectivity values are plotted in Fig. 16(a) and Fig. 16(b), respectively. As illustrated, the present formulation can accurately predict the theoretical results (see p. 27, Ref. 10).

### 3.4. *Pressure attenuation of an acoustic expansion muffler*

The above analysis example verifies that the present $\mathbf{u}/p$ mixed finite element formulation can simulate the absorptive phenomenon accurately. To test this feature further, pressure attenuation of an acoustic muffler is compared with the analytical pressure attenuation formula.[8] The characteristics of this two-dimensional expansion muffler without fibrous material, including the analytical transmission-loss (TL) curve, are well studied.[8] The incoming wave pressure was first calculated using Eq. (28):

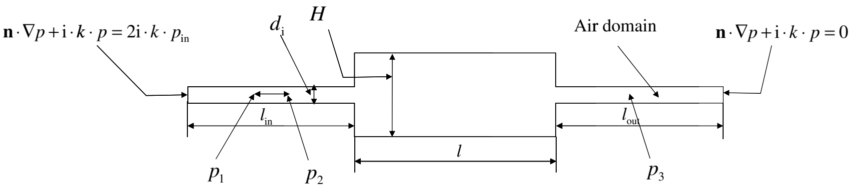
$$p_i = \frac{1}{e^{ikx_1}} \frac{p_1 - p_2 e^{-ikx_{12}}}{1 - e^{-i2kx_{12}}}, \quad (\text{where } p_1 = p_i e^{ikx_1} + p_r e^{-ikx_1},\ p_2 = p_i e^{ikx_2} + p_r e^{-ikx_2}), \quad (27)$$

where $p_1$ and $p_2$ are the complex pressures at the arbitrary points of the inlet, and the distance between the two points ($x = x_1$, $x = x_2$) is $x_{12}$. The magnitudes of the incoming and reflected waves are denoted by $p_i$ and $p_r$, respectively. The TL of the acoustic muffler is then approximated by the three-point method that calculates the ratio of the incoming wave pressure to the transmitted wave pressure such that:

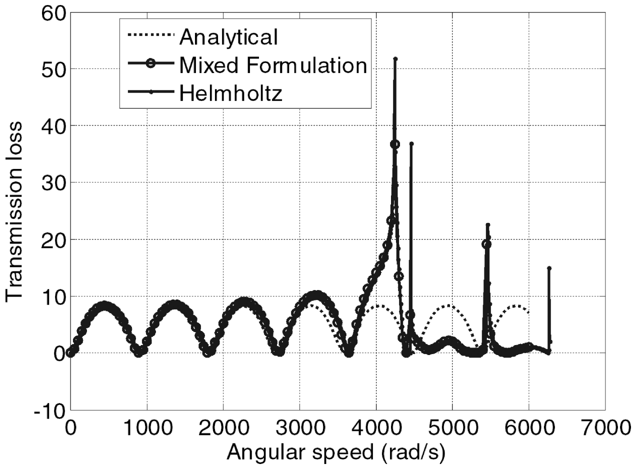
$$TL = 20 \log\left(\left|\frac{p_i}{p_3}\right|\right) + 10 \log\left(\left|\frac{A_i}{A_o}\right|\right), \quad (28)$$

where the complex pressure output at the outlet is denoted by $p_3$. The areas of the inlet and outlet tubes are denoted by $A_i$ and $A_o$, respectively.

First, without fibrous or solid media, the simple expansion chamber is modeled by the present mixed FE formulation, and its TL responses are calculated with respect to the excitation frequencies in Fig. 17. As shown, the accurate calculation of the TL values is possible.

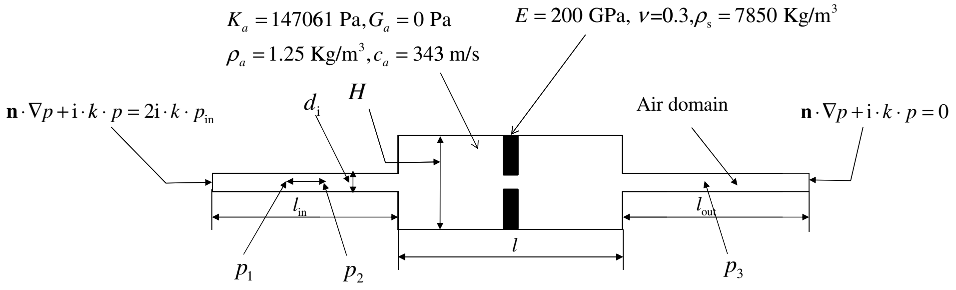



(a)

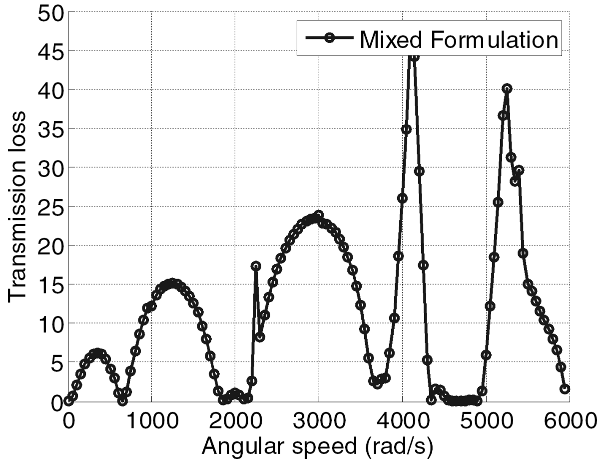



(b)

Fig. 17. Example of 2D muffler analysis[8]: (a) muffler geometry ($\rho_a = 1.25\,\mathrm{kg/m^3}$, $c_a = 343\,\mathrm{m/s}$, $l = 1.2\,\mathrm{m}$, $l_{\mathrm{in}} = l_{\mathrm{out}} = 1.0\,\mathrm{m}$, $H = 0.5\,\mathrm{m}$, $d_i = 0.1\,\mathrm{m}$, and $x_{12} = 0.1\,\mathrm{M}$); (b) transmission-loss curves of 1D analytical TL, TL by the Helmholtz equation, and TL by the present mixed formulation.




(a)



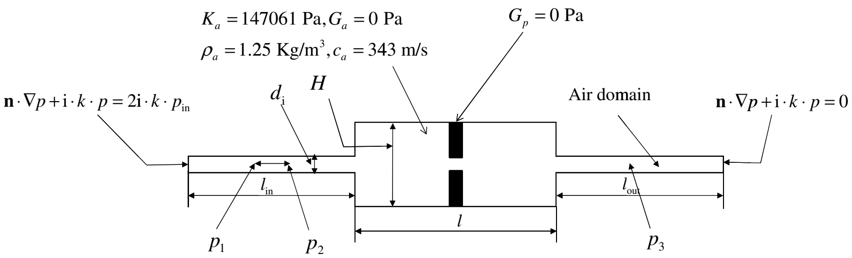

(b)

Fig. 18. Example of 2D muffler analysis with the solid bar inside the middle of the expansion chamber: (a) muffler geometry (solid bar with dimensions of $0.02\,\mathrm{m} \times 0.2\,\mathrm{m}$); and (b) transmission-loss curves of solid bar inside the middle of the expansion chamber.

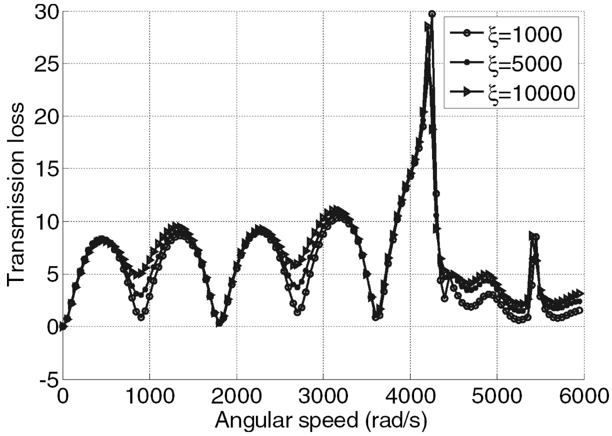
Note that after 3500 rad/s, some differences were observed in the TL values of each method because the analytical TL value from Eq. (29) is derived by a one-dimensional assumption. Therefore, for higher excitation frequencies, the one-dimensional assumption is violated. Second, in Figs. 18 and 19, the elastic box (whose material properties are those of aluminum) and the fibrous box are posed inside the chamber. Because of the inclusion of the elastic and fibrous boxes, the different pressure attentions can be obtained. Compared with the TL of an air-filled chamber, the different impedance curve with the aluminum bars is improved in Fig. 18. For the fibrous bars with different flow-resistivity values, the higher TL curves can be obtained in Fig. 19 as expected. These numerical examples again illustrate that the present unified mixed FE formulation can simulate the acoustic-porous-structure interaction.

### 3.5. *Eigenfrequency analysis*

For the last numerical example, the eigenfrequency of the rectangular box is calculated with the present mixed formulation in Figs. 20 and 21. With the assumption of the pure acoustic

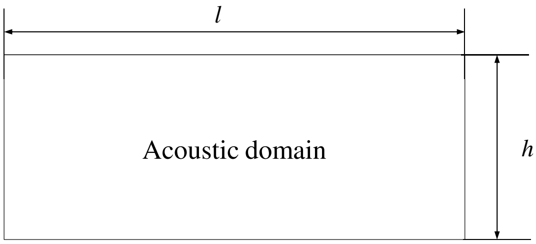



(a)

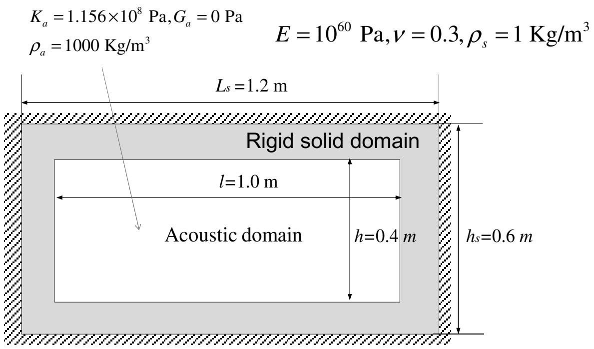



(b)

Fig. 19. Example of 2D muffler analysis with the fibrous bar inside the middle of the expansion chamber: (a) muffler geometry (fibrous bar with dimensions of $0.02\,\mathrm{m} \times 0.2\,\mathrm{m}$, $\xi = 1000\,\mathrm{Nm}^{-4}\mathrm{s}$, $\xi = 5000\,\mathrm{Nm}^{-4}\mathrm{s}$, and $\xi = 10\,000\,\mathrm{Nm}^{-4}\mathrm{s}$); and (b) transmission-loss curves with different flow-resistivity values.




(a)




(b)

Fig. 20. Eigenfrequency analysis using (a) Helmholtz equation and (b) mixed formulation with stiff elastic foundation.

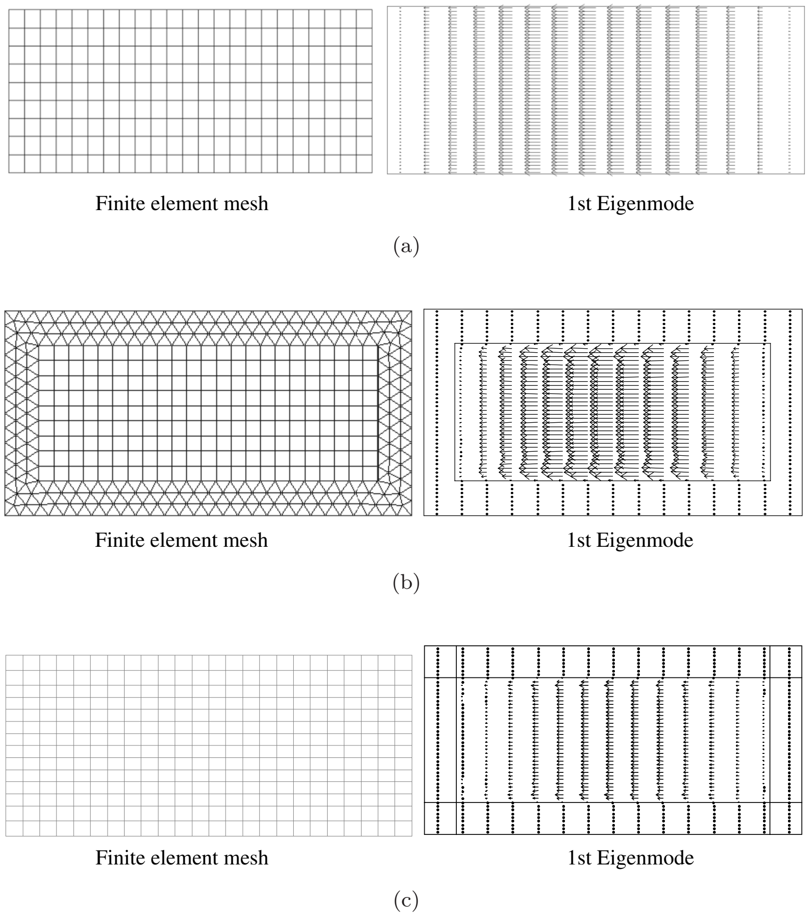



Fig. 21. Pressure plots of Helmholtz's equation and mixed finite element formulation. (a) Helmholtz equation (170 Hz), (b) Mixed formulation (167.77 Hz) and (c) Mixed formulation with rectangular meshes (167.9789 Hz).

equation or Helmholtz's equation, fluid near the walls moves parallel to the walls, and the normal velocity component is zero. However, in the mixed formulation, the fluid velocities near the walls are assumed to zero (Fig. 21), which is one of the notable differences between the Helmholtz's equation and mixed formulation. At this point, it should be mentioned that the Helmholtz's equation is the simplest equation for sound in fluid after assumptions such as no loss and small density changes. The mixed formulation considers more complex redundant fluid velocities. As a result, the first eigenvalue of the mixed formulation is lower than that of the pure Helmholtz's equation. Furthermore, using the triangular mesh, spurious modes of the mixed finite element in the eigenfrequency analysis were observed. Thus, quadrilateral meshes for the acoustic domain were used in this study. Because the eigenfrequencies of the solid domain in Fig. 21(b) are much higher than those of the first mode of the acoustic domain, the first mode of the acoustic domain was obtained even with the triangular mesh at the rigid solid domain.

## 4. Conclusions

This study has developed a new monolithic mixed finite element formulation for an acoustic-porous-structure interaction system. The present finite simulation with the mixed formulation provides a means for rapid and easy consideration of the coupling effects of the media. To model the pressure attenuation of fibrous material, the Delany–Bazley material model is implemented rather than the phenomenological model. The main benefit of employing the empirical model is that the complicated pressure propagation behavior of fibrous material can be easily considered with a few parameters of fibrous material. The empirical material model is generally employed and formulated in the acoustic equation (i.e. the Helmholtz equation); however, this research reveals that the empirical material model can also be used inside the mixed formulation. The accuracy and efficiency of the present method are demonstrated through several analysis examples in which the wall and part of an analysis domain are partially treated with sound-absorbing material. In future research, a structural topology optimization for fibrous, acoustic, and structure interaction is planned.

## Acknowledgments

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