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Tailoring a bidirectional negative stiffness (BNS) structure with mechanical diodes for mechanical metamaterial structures

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Abstract

A new structure with a bidirectional negative stiffness (BNS) value utilizing buckling phenomena (often called bi-stable or snap-through) and a mechanical diode are presented with regard to mechanical metamaterial applications. The need for cost and mass efficient vibration isolation parts within the modern aerospace, automotive, and civil industries has been the subject of many interesting studies in the last several decades. With conventional materials in nature, many innovative approaches have been proposed, but with some limitations for vibration suppression. To cope with these difficulties, it was possible to employ a man-made material with a negative density or negative stiffness, not existing in nature. Recently metamaterials, i.e., man-made repeating structures with negative densities or/and stiffness values, have been proposed and their applications have been reported. This research presents a new man-made structure with a BNS, using the help of bi-stable mechanisms and a mechanical diode. With regard to the metamaterial application of this phenomenon, the dispersion nature of a 1D bi-stable mechanism has been studied and a new novel structure having BNS with a mechanical diode is presented. The effects of states of snap-through were highlighted and quantified experimentally.

Keywords: meta-material, negative spring, mechanical diode, bi-stable mechanism

(Some figures may appear in colour only in the online journal)

1. Introduction

This study presents a new mechanical system with a mechanical diode, exhibiting a bidirectional negative stiffness (BNS) value. Many relevant studies regarding the engineering applications of filters, resonators, and waveguides, have exploited the particular filtering properties of a periodic structure or a material with a periodic microstructure (Klatt and Haberman 2013, Zheng *et al* 2014). This phenomenon has been extensively studied in the last thirty years in the fields of optics, acoustics, and solid mechanics (Fang *et al* 2006, Nicolaou and Motter 2012, Fulcher *et al* 2014). In the present study, a mechanical structure was devised with a bi-stable mechanism and exhibiting a BNS value. Since it is difficult to achieve a negative stiffness (NS) in two directions, a combination of two bi-stable mechanisms with new mechanical diodes is presented. The novel mechanisms exhibit very peculiar characteristics that highlight them as interesting candidates for the realization of very sensitive filters and insulators with regard to mechanical engineering applications.

1.1. Metamaterial

Man-made periodic structures have demonstrated interesting filtering properties for mechanical, electromagnetic, and acoustic waves. These tailored properties can be exploited in many scientific and engineering applications for filters, resonators, and waveguides (Wang and Lakes 2004, Florijn *et al* 2014). Their unusual effective parameters, not observed in nature, have led to numerous interesting and remarkable applications, such as: cloaking, focusing and imaging, non-reciprocal transmission, wave propagation and wave front engineering, superlenses, and hyperlenses (Lee *et al* 2007, Le and Ahn 2011). Recently, most presented metamaterials are

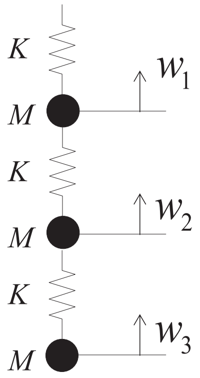

Figure 1. Repeating mass–spring system.

effective within a certain frequency range and are hardly adjustable after fabrication. Thus, it is not possible to use these materials in dynamically changing real-world applications. One possible solution to overcome this issue involves the incorporation of active designs with vibrational characteristics that can be controlled (Lakes *et al* 2001, Lakes and Drugan 2002). In the present study, we proposed a new structure composed of two bi-stable mechanisms and a mechanical diode, which can be incorporated with a mechanical metamaterial.

1.2. Mechanical metamaterials and NS structures

Despite the recent metamaterial developments described above, periodic mechanical structures have rarely been investigated with regard to structural mechanics. To understand the characteristics of mechanical metamaterials, consider the mass–spring system in figure 1. With the repeated spring–mass structure which is used as an introductory example for the metamaterial, however, it shows the very interesting mechanical properties due to its periodicity. Not to mention, the spring constant and the mass are assumed to be positive values. In our research, we were interested in cases where one becomes negative, either negative mass or NS. From a physics perspective, negative density is a topic linked to the Nobel Prize. The topic of negative springs is also a Nobel Prize subject because negative springs may violate conservation of energy. Not to mention the presented mechanism does obey the conservation of energy and the energy stored in the mechanism is used to show the NS behavior for external load; some typical example systems with NS values can be seen in figure 2 (a bi-stable hairpin and rigid-body mechanisms with springs). The subject of the NS behavior is not a new topic and many relevant studies utilizing buckling phenomena can be found in textbooks.

When a body is infinite, the effect of periodicity on its dynamic behavior can be investigated by considering only the elementary cell of the problem and applying the Floquet–Bloch theorem. The analysis result is a dispersion diagram displaying a range of frequencies associated with waves propagating along the body (pass bands) alternating with a range of frequencies corresponding to waves that cannot be transmitted (stop bands or band gaps). For a finite structure, analysis is based on an evaluation of the transmission coefficient that measures energy transmission through a periodic body from a signal source to a control point.

In this paper, the design of a BNS structure comprising a mechanical diode was initially exhibited and its experiment results are presented. With bi-stable mechanisms, it is often possible to demonstrate one-directional NS behavior as shown in figure 2. However the application of the NS behavior is intricate because its values vary and it is difficult to maintain the states with NS values. Furthermore, it is difficult or impossible to demonstrate BNS behavior. This research presents an innovative structure composed of two bi-stable mechanisms and some auxiliary mechanical diodes posing as two bi-stable mechanisms in the NS region. This BNS structure can be employed as an important unit cell in figure 1. To our knowledge, the BNS structure has not been proposed. Theoretical and experiment results suggest its potential application to significantly increase the inherent damping ratio of an oscillator.

The layout of this paper is as follows. After describing the equations related to the theory of wave propagation in a 1D periodic structure, the concept of an NS structure using compliant bi-stable mechanisms is presented and a unit cell of the BNS mechanism is manufactured. Using a force measurement system, responses are measured. The conclusion section summarizes our findings and presents additional research topics.

2. Theory of wave propagation in a 1D periodic structure

Before presenting our novel mechanism with NS, wave propagation theory in a 1D periodic structure is presented. The dispersion and modal band structure of nonlinear waves is assumed to be lacking a material gradient through its thickness. A schematic with 1D periodic springs and mass and a basic unit cell with a periodicity length of a can be seen in figure 1.

2.1. Wave propagation in metamaterials

Accordingly, the general modal solution for the displacement w of a 1D periodic structure with a lattice constant of a could be defined by the harmonic modulation of an a periodic field w_p as follows:

$$w(x, y, t) = w_p(x, y)\mathrm{e}^{\mathrm{i}(ky-\omega t)}, \;\; w_p(x, y) = w_p(x, y + a), \tag{1}$$

where ω is the angular frequency and k is a wave number for the y-direction; the rectangular structure is considered a mass by neglecting its deformation.

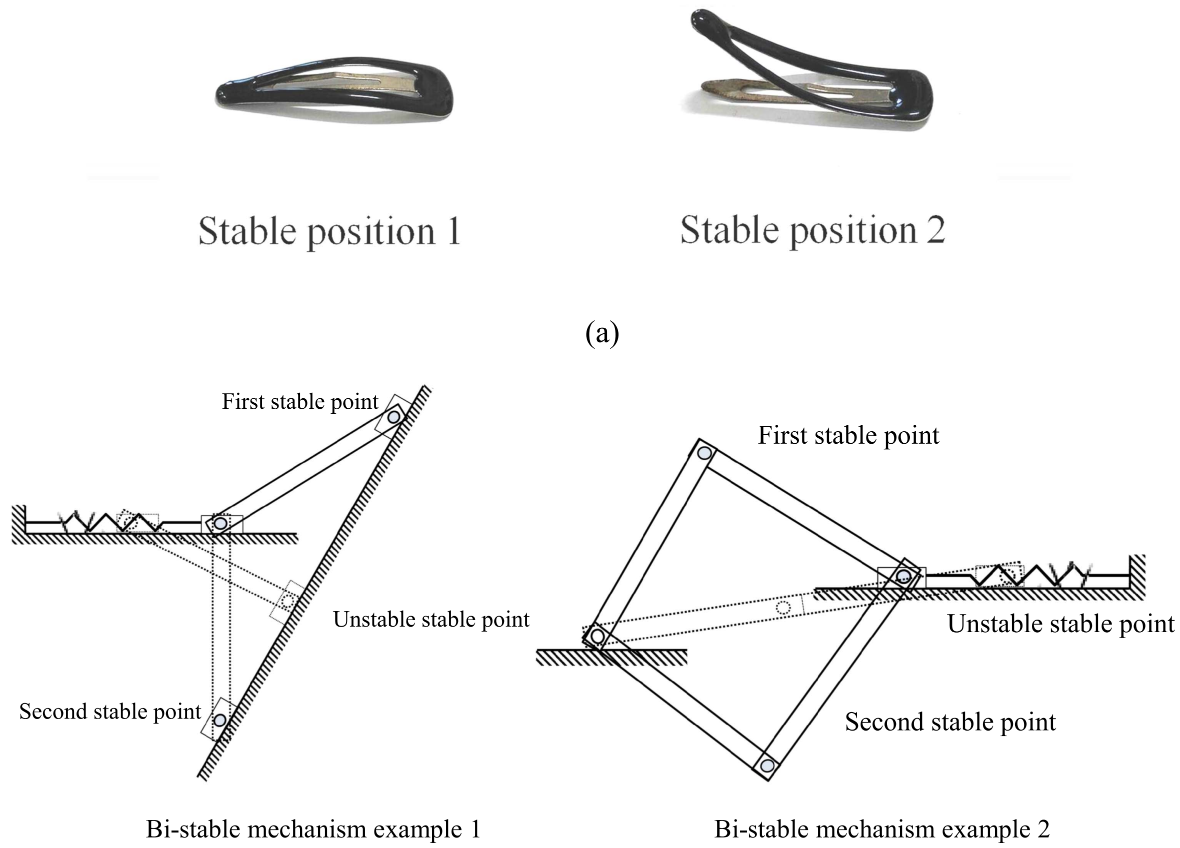

Figure 2. Bi-stable mechanism examples (a) bi-stable hairpin and (b) bi-stable mechanism examples.

With the Bloch–Floquet condition, the following dynamic system can be obtained:

$$[K^t(k) - \omega^2 M]\mathbf{w}e^{i(ky-\omega t)} = \mathbf{0},\ \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}. \tag{2}$$

The tangent stiffness matrix K^t (or K) is complex and dependent on the wave number k. Due to the periodicity of the structure, a representative unit cell is modeled with its length equal to the lattice constant a. Finally, the modal response of the unit cell is obtained by eliminating $e^{i(ky-\omega t)}$; the eigenvalue analysis for non-trivial solutions is thus formulated as follows:

$$|K_p^t(k) - \omega^2 M_p| = 0, \tag{3}$$

where K_p^t and M_p are the reduced tangent stiffness and mass matrices obtained after applying the periodic boundary condition. A gradient of eigenvalues versus wave number k is determined, giving the band structure of a periodic unit cell. Principally, k can possess any value but due to periodicity in the Bloch–Floquet condition, the value is limited to the first Brillouin zone (a uniquely defined primitive cell in reciprocal space), $-\frac{\pi}{a} \leqslant k \leqslant \frac{\pi}{a}$, and further to the irreducible Brillouin zone, $0 \leqslant k \leqslant \frac{\pi}{a}$.

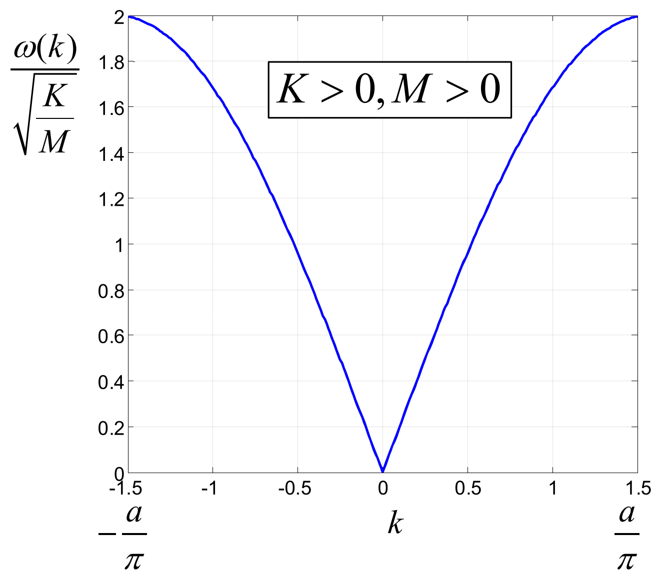

Figure 3. Dispersion curve with positive stiffness.

2.2. 1D dispersion relationship $\omega(k)$ for a mass–spring chain

Before further analyzing the vibrations of a metamaterial with a negative spring constant, this subsection revised the vibration of an infinite one-dimensional linear chain of masses with an identical mass M, connected via springs

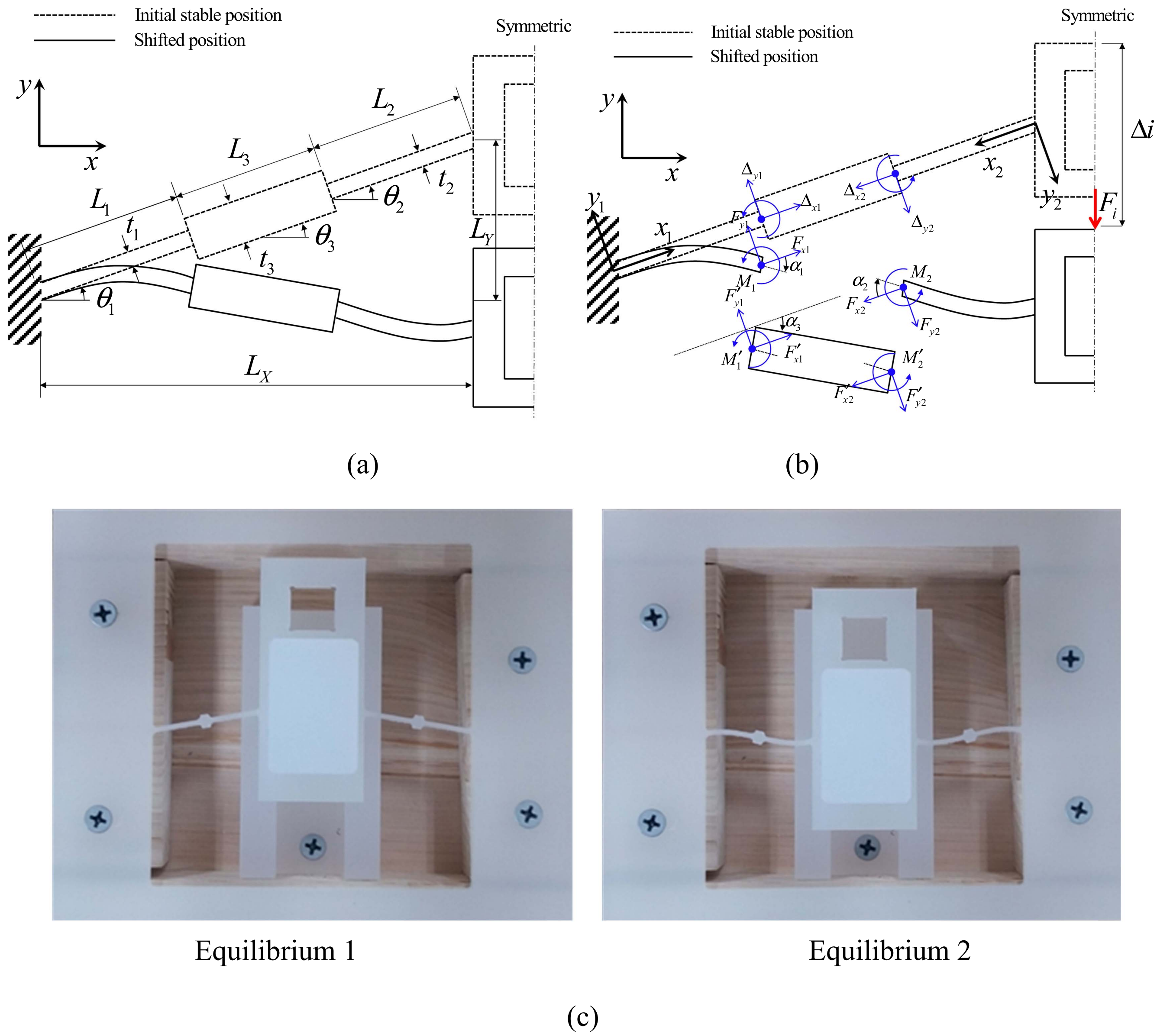

Figure 4. Fully compliant bi-stable mechanism. (a) Schematic illustration, (b) free body diagram and (c) actual two equilibrium states. (n: 2, w: 6 mm, L_1: 15 mm, θ_1: 10°, t_1: 1.2 mm, L_2: 15 mm, θ_2: 10°, t_2: 1.2 mm, L_3: 2 mm, θ_3, 10°, E: 1.42 GPa, v: 0.33, t_3: 3.6 mm).

with a constant spring constant, K, as seen in figure 3. The distance between the two masses in their equilibrium position was a; w_i was the displacement of the ith mass from this equilibrium position.

The Newton equation of motion for this system is written as follows:

$$M\frac{\mathrm{d}^2 w_i}{\mathrm{d}t^2} = K(w_{i-1} - w_i) - K(w_i - w_{i+1}). \tag{4}$$

Assuming harmonic motion dependence $\mathrm{e}^{-\mathrm{j}\omega t}$ and using Bloch's condition, $(w_{i+1} = w_i \mathrm{e}^{\mathrm{j}ka},\ w_{i-1} = w_i \mathrm{e}^{-\mathrm{j}ka})$, the following conditions can be derived below, in which the wavenumber is k.

$$-\omega^2 M w_i = K(w_{i-1} - 2w_i + w_{i+1}), \tag{5}$$

$$-\omega^2 M w_i = K(\mathrm{e}^{-\mathrm{j}ka} - 2 + \mathrm{e}^{\mathrm{j}ka})w_i, \tag{6}$$

$$\begin{aligned}\omega^2 M w_i &= 2K(1 - \cos(ka))w_i\\ &= 4K \sin\left(\frac{ka}{2}\right)^2 w_i.\end{aligned} \tag{7}$$

Comparing the above equation with the dispersion relation of an infinite mass–spring vibration, the effective mass and effective stiffness can be defined as follows:

$$M_{\mathrm{eff}} = M,\ \ K_{\mathrm{eff}} = 2K(1 - \cos(ka)) = 4K\sin\left(\frac{ka}{2}\right)^2. \quad (8)$$

From equation (7), the following conditions are computed:

$$\cos(ka) = \frac{2K - M\omega^2}{2K}, \quad (9\text{-}1)$$

$$\omega(k) = 2\sqrt{\frac{K}{M}}\left|\sin\left(\frac{ka}{2}\right)\right|. \quad (9\text{-}2)$$

From equation (9-1), it can be seen that ka should be an imaginary value for certain frequency ranges where:

$$\frac{2K - M\omega^2}{2K} > 1 \text{ or } \frac{2K - M\omega^2}{2K} < -1. \quad (10)$$

However, since $M\omega^2$ and K are positive values for the normal system, the first relationship (10) can not be satisfied and only the second relationship can be considered. In other words, the imaginary wave speed can be achieved in a periodic structure for the following frequency ranges:

$$\frac{2K - M\omega^2}{2K} = 1 - \frac{M\omega^2}{2K}\langle -1 \rightarrow \omega \rangle 2\sqrt{\frac{K}{M}}$$
$$\text{(with positive } M \text{ and positive } K \text{ value).} \quad (11)$$

However, in the present nonlinear structure based metamaterial, the stiffness value K can be negative and the first relationship in (10) could be satisfied.

$$\frac{2K - M\omega^2}{2K} > 1 \text{ (with positive } M \text{ and negative } K \text{ value).} \quad (12)$$

One of the interesting points in (12) is that evanescent wave can be achieved for all frequency ranges.

3. Tailoring a NS matrix using a compliant bi-stable mechanism

3.1. Fully compliant bi-stable mechanism

Figure 2 shows a few typical bi-stable mechanisms, i.e., a bi-stable hairpin and two rigid-body mechanisms with an elastic spring, which are widely used in our daily life without recognition. To realize this bi-stable mechanism, the buckling phenomenon of a beam can be employed (Jensen and Howell 2003, 2004, Masters and Howell 2003, Sonmez and Tutum 2008). For example, figure 4 shows a bi-stable mechanism consisting of two thin beams and a thick center beam. Due to the buckling phenomena of two beams, the two equilibrium positions in figure 4 can exist in which external forces are in balance with internal forces. It has been reported

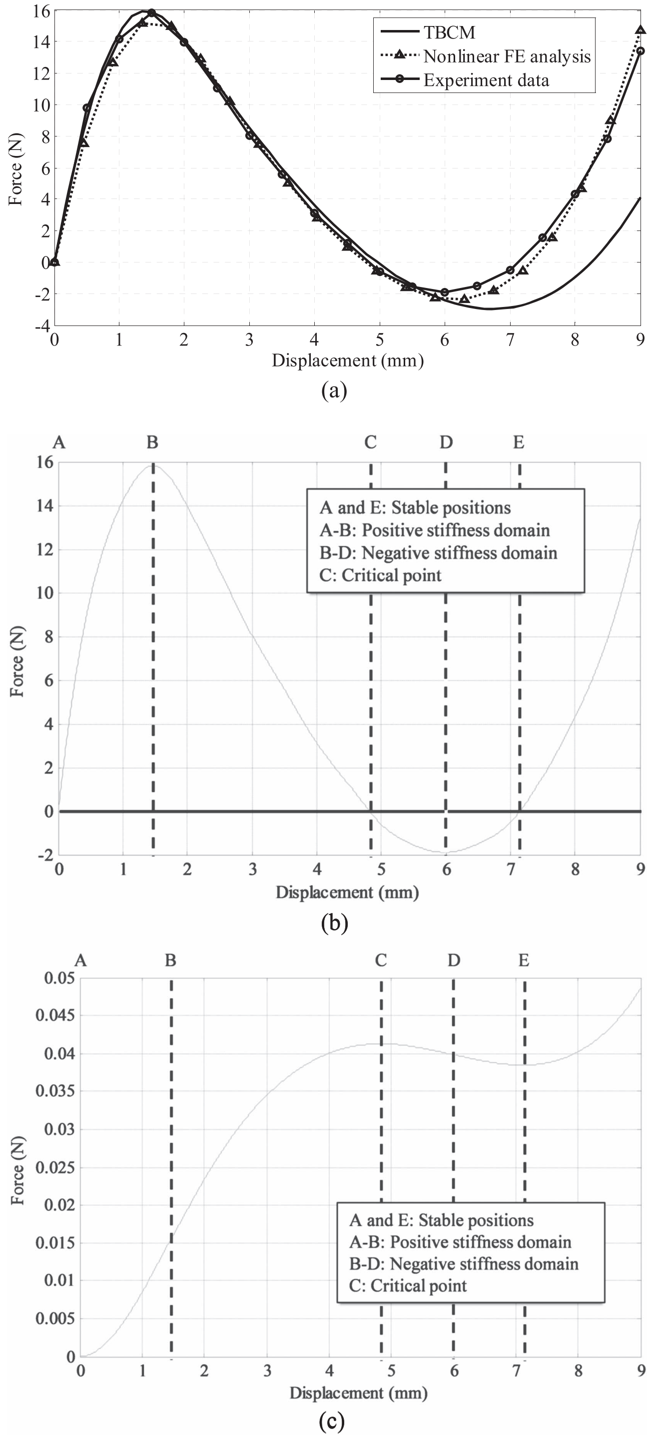

Figure 5. (a) The kinetostatic behavior of the bi-stable mechanism in figure 4 and (b) the potential energy curve. (A: 0 mm, B: 1.46 mm, C: 4.82 mm, D: 6 mm, E: 7.16 mm).

that due to its interesting properties, this bi-stable mechanism is suited for robotic applications (Jensen *et al* 1999, Qiu *et al* 2004, Hansen *et al* 2007, Chen and Zhang 2011). The rapid physical change in configuration between the two equilibrium states is sometimes referred to as a snap-through phenomenon (Howell 2001, Wilcox and Howell 2005). From a mechanical analytical perspective, this mechanism requires

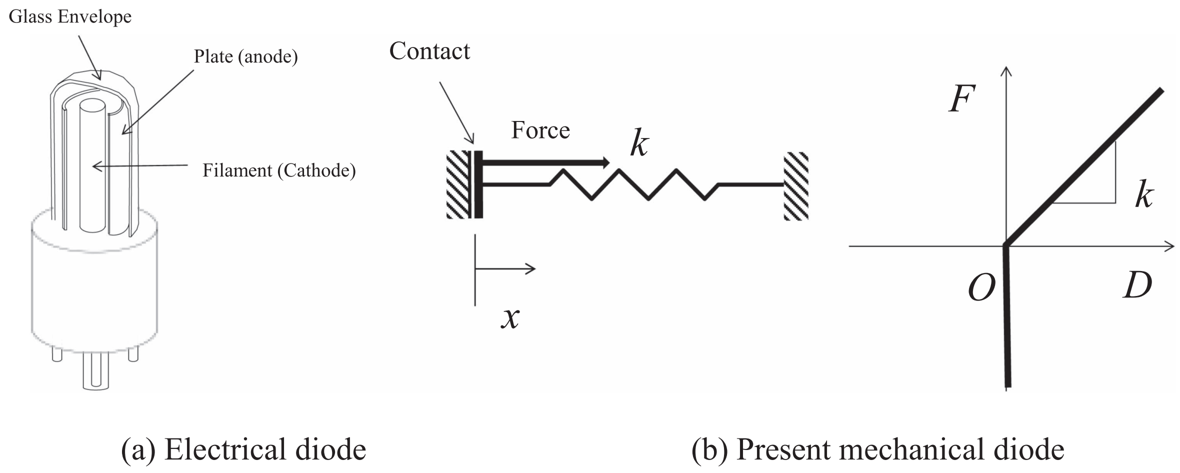

Figure 6. The present mechanical diode concept: (a) electrical diode and (b) present mechanical diode.

geometrical nonlinear analysis to analyze the buckling phenomenon (Hwang *et al* 2003, Jutte and Kota 2008, Pucheta and Cardona 2010, Zhang and Chen 2013); geometrically nonlinear Timoshenko beam theory can be applied to analyze this bi-stable mechanism. The actual two equilibrium states are illustrated in figures 4(a), (b) and our experiment can be seen in figure 4(c).

With the help of Timoshenko beam theory in assuming the center rectangular structure to be a rigid box, the Y-displacement and applied force are expressed as follows (see Awtar *et al* 2007, 2010 for additional details):

$$\begin{Bmatrix} L_X \\ L_Y - \Delta i \end{Bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{Bmatrix} L_1 + \Delta x1 \\ \Delta y1 \end{Bmatrix} + L_3 \begin{Bmatrix} \cos(\theta_3 + \alpha_3) \\ \sin(\theta_3 + \alpha_3) \end{Bmatrix} - \begin{bmatrix} \cos(\pi + \theta_2) & -\sin(\pi + \theta_2) \\ \sin(\pi + \theta_2) & \cos(\pi + \theta_2) \end{bmatrix} \times \begin{Bmatrix} L_2 + \Delta x2 \\ \Delta y2 \end{Bmatrix}, \tag{13}$$

$$F_i = F_{x2}\sin(\pi + \theta_2) + F_{y2}\cos(\pi + \theta_2), \tag{14}$$

where the representative displacement and external forces are denoted by Δi and F_i. The other terms can be found in figure 4 (see Timoshenko 1976, Hutchinson 2001, Chen and Ma 2015 for additional details). If a structure possesses n ribs of the above beam, the total input force can simply be summed as follows:

$$F_{\text{total}} = nF_i, \tag{15}$$

where the number of ribs is n and the total force is denoted by F_{total}.

In order to test and illustrate interesting characteristics, some polypropylene samples are manufactured with a CNC milling machine with ±0.05 mm tolerance levels, as shown in figure 4. To measure the magnitude of force, a load cell compatible with 20 kgf is installed and the force versus displacement data is continuously recorded. For precise movement, displacement at the loading point is raised or lowered with a 0.5 mm accuracy with the positioning station. The theoretical and experimental force curves and displacement are compared in figure 5(a) and are in good agreement. In our experiments, we can repeatedly obtain the similar responses with the samples made with polypropylene; some cares should be made at the corners of the structure not to have the stress concentrations and the plastic deformations. Unlike linear structures exhibiting a linear relationship between force and displacement, a nonlinear relationship between force and displacement can be achieved. It is important to notice that NS between 1.3 and 5.8 mm could be observed. Figure 5(b) shows the potential energy of the bi-stable mechanism, proving that it does not violate any laws of physics. In other words, the stored energy in the bi-stable mechanism is used to show this interesting NS relationship. In figure 5(b), the two equilibrium positions are indicated by 'A' and 'E'. The first equilibrium at 'A' is the point with 0 N and 0 mm and the second equilibrium point at 'E' is at 0 N and 7.1 mm. The tipping point or the critical point around 4.8 mm is an important point because the mechanism moves to the first equilibrium state prior to the that tipping point and moves to the second point after the tipping point. The approximated stiffness value of the present bi-stable mechanism is around $-4\ \text{N mm}^{-1}$ (NS value).

4. Unit cell for BNS with a mechanical diode

As noticed in sections 1 and 2, a metamaterial structure with an NS value possessed a very interesting feature and it

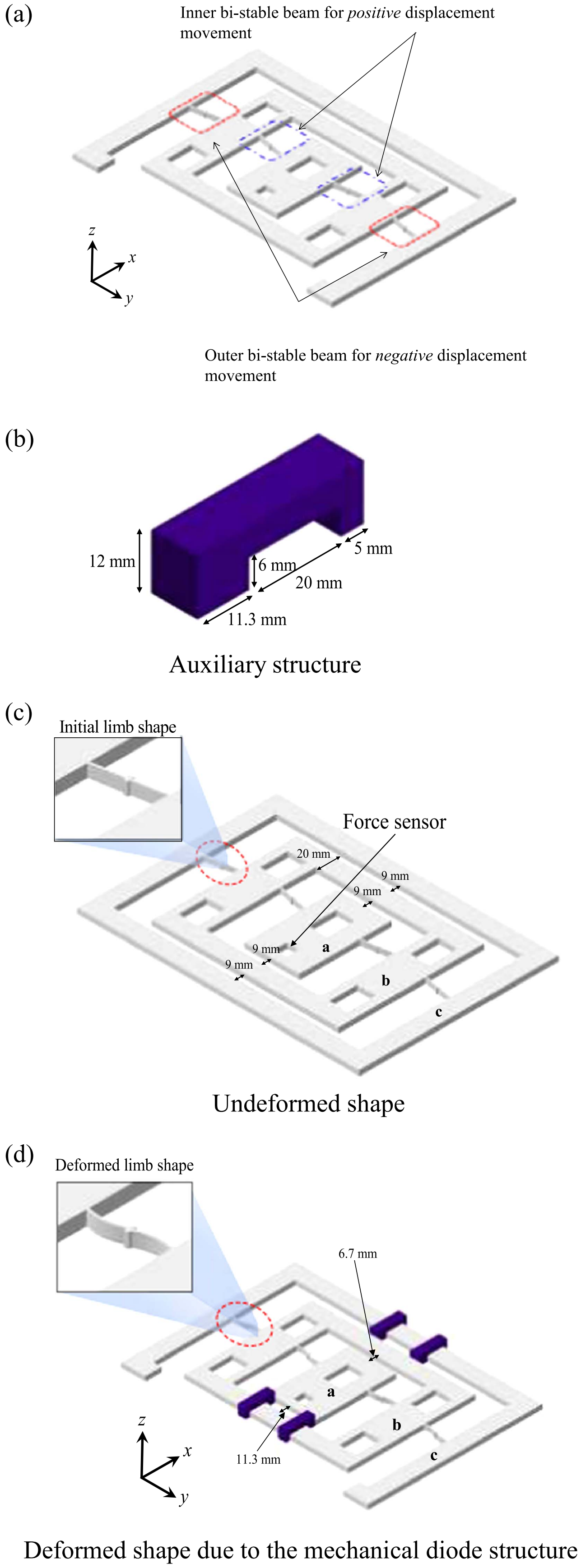

Figure 7. The present unit cell for the BNS structure.

became obvious to apply this bi-stable mechanism to a metamaterial structure. However, because the bi-stable mechanism was unstable between the two stable points, it was impossible to apply the system for the purpose of vibration suppression. Thus, the following observations were made.

Observation (1) under preloading conditions, displacement or force can be applied to move the bi-stable mechanism into the region with an NS value. Under force loading conditions, it was hard to maintain its status in the NS region.

Observation (2) depending on the preloading conditions, the structure suddenly switched toward one of the two equilibrium states. This prohibited us from using the bi-stable mechanism in the region with NS values.

Observation (3) due to instabilities, it was hard to use the bi-stable mechanism with an NS value.

Observation (4) special ideas are required to maintain the bi-stable mechanism with NS values during operation, i.e., vibration.

From the above observations, it becomes our proposition to devise a new system with the concept of a mechanical diode. In other words, the preloading conditions in both directions can be realized with a mechanical diode.

4.1. Mechanical diode

In electronics, a ‘resistor’ is a two-terminal electronic component producing a voltage across its terminals that is proportional to the electric current passing through it in accordance with Ohm’s law, i.e., Voltage = Resistance × Current. Interestingly it is similar to the mechanical spring, i.e., Force = Stiffness × Displacement. Therefore, it is conventional to exploit this similarity for an electronic engineer to understand the properties of a mechanical system and vice versa. In electronics, there exist ‘diodes,’ whose function is to allow an electric current to pass in one direction. Ideally, diodes possess a low (ideally zero) resistance to the flow of current in one direction, and high (ideally infinite) resistance in the other direction. The realization of a mechanical diode is not obvious; one potential realization involved using an extremely brittle material with different stiffness values under either tension or compression. In the present study, we notice the application of a contact mechanism in order to realize the mechanical diode as shown in figure 6.

4.2. BNS structure

After some trial and error with the mechanical diode concept, we can present the following mechanism containing two bi-stable mechanisms in figure 7. Note that the inner two bars are marked with dotted circles to yield NS values for movement in the positive direction and the inner two bars were marked with circled lines to yield NS values for movement in the negative direction in figure 7(a).

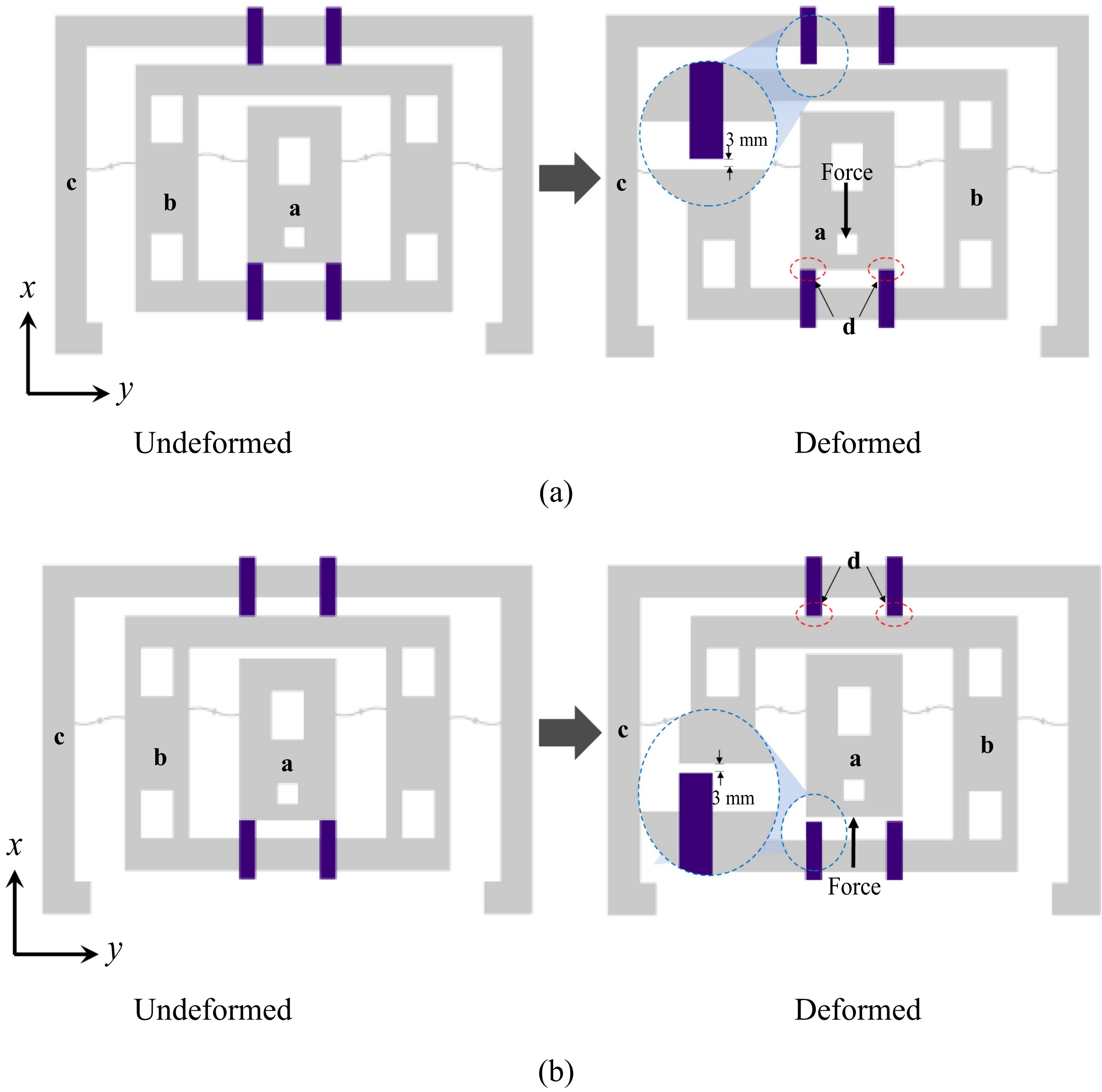

Figure 8. Mechanical deformation with positive and negative forces: (a) mechanical deformation with a negative directional force and (b) mechanical deformation with a positive directional force.

Furthermore, there are the four auxiliary structures causing structural preloads for the realization of the mechanical diode in figure 6(b). With the four mechanical diode structures in figure 7(d), the four ribs can be preloaded and the global stiffness of the mechanism can yield NS values in the both directions.

To make it clear, figure 8 shows exaggerated deformation shapes of the bi-stable mechanism. First, let us assume a negative load in figure 8(a). Due to geometry restrictions of the internal rectangular box (structure 'b' in figure 8(a)) near the lower auxiliary supporting structures, the two ribs connecting structures (a) and (b) are loaded. Prior to the application of force, the displacement of structure 'a' is preloaded due to the upper mechanical diode structure having an NS value. Therefore, the relationship between the externally applied load and the displacement exhibit an NS value. On the other hand, let us assume a positive load in figure 8(b). The opposition phenomenon is observed.

4.3. Working principle of the BNS structure

Due to the upper mechanical diode structure moving together with the structure 'b', the external applied force yields a deformation to the two ribs connecting the structures 'b' and 'c'. Note that due to the lower mechanical diode structures, the two libs are already loaded in the NS regions. Conclusively, depending on the direction of the applied load, the ribs with NS are switched. The overall stiffness of

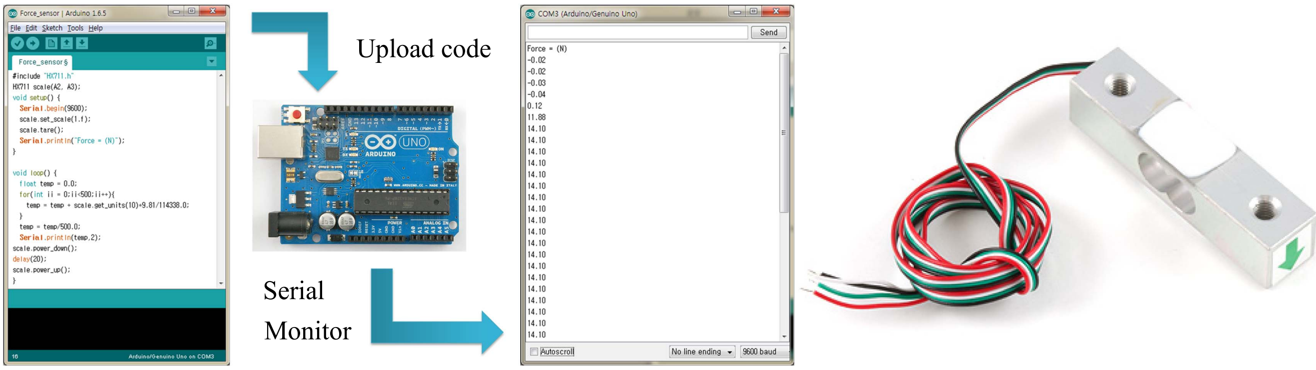

Figure 9. Measurement system using an Arduino system and force sensor.

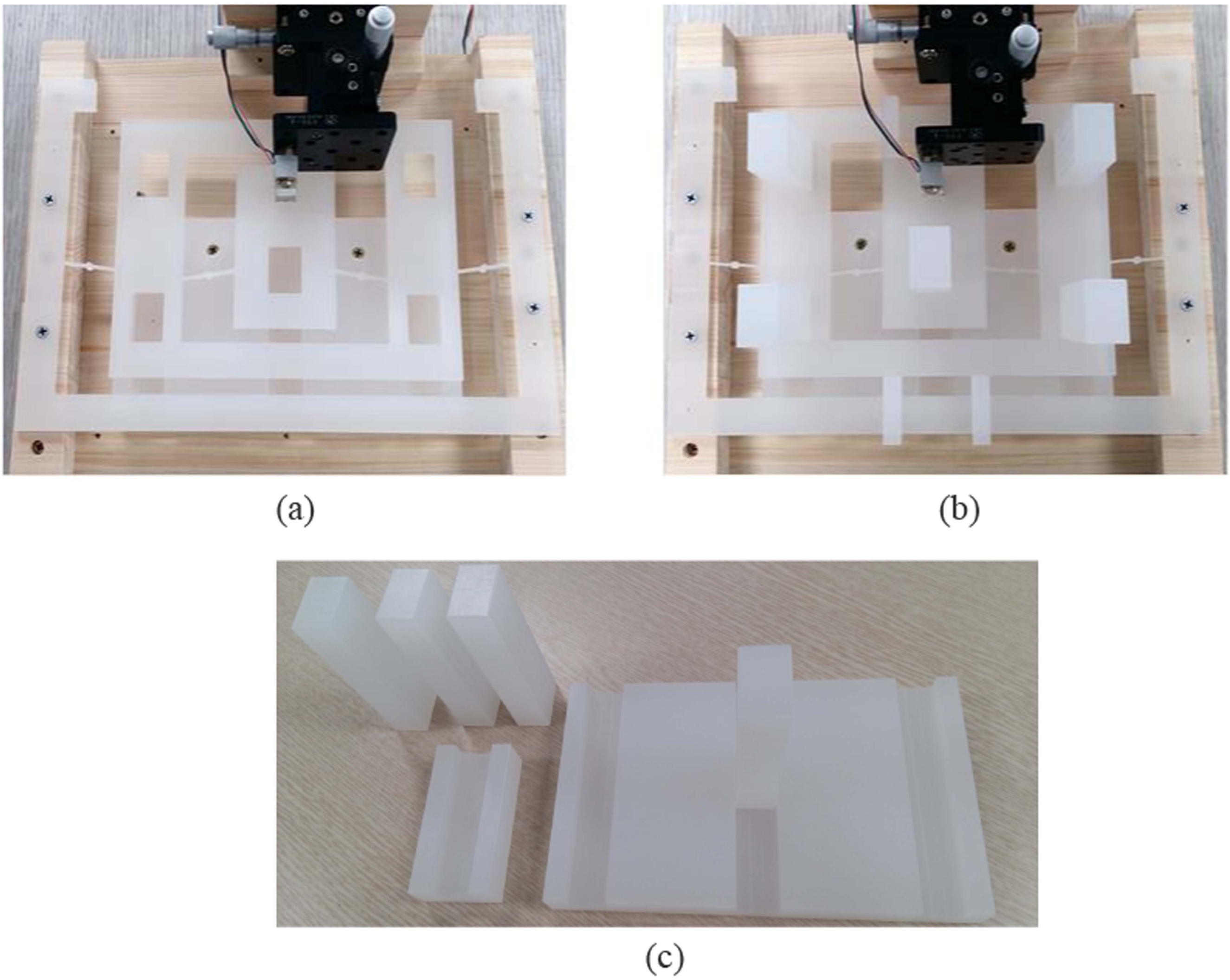

Figure 10. Unit cell with bi-directional negative stiffness. (a) A unit cell, (b) an installed unit cell with a mechanical diode and (c) a guide mechanism.

the present structure becomes negative. An equivalent spring network is drawn in figure 11(a) and its energy graph in figure 11(b).

4.4. Experiment and measurements

In order to verify the diode mechanism concept above, the following polypropylene structure in figure 10 is created with

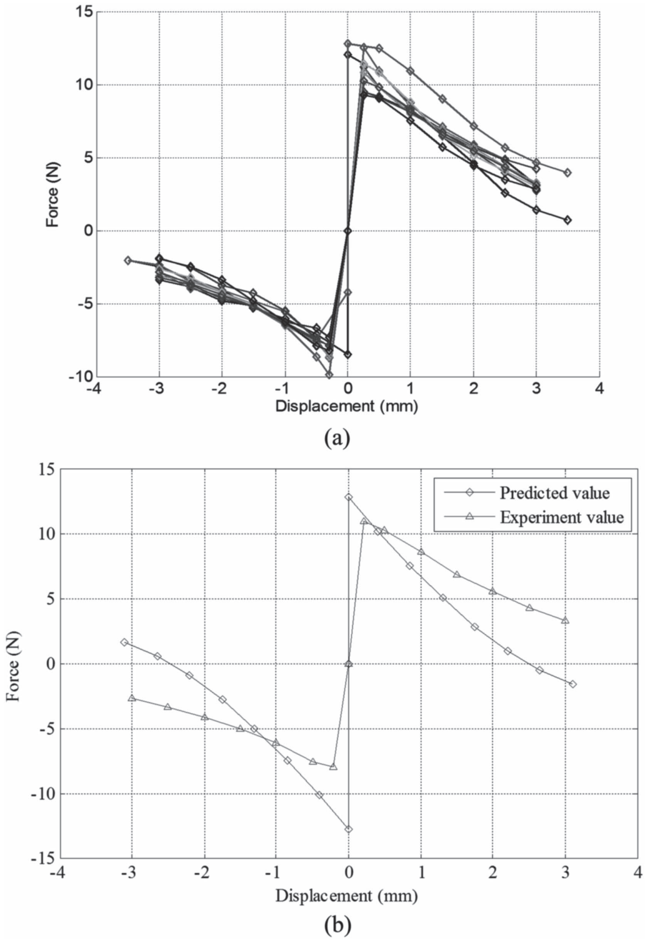

Figure 11. Force and displacement curve of the present bi-directional mechanism. (a) Experiment data (10 times) and (b) the theoretical curve and the averaged experiment curve.

a CNC machine. The main structure in figure 10(a) possessed five vertical bars. These vertical bars are used to guide or constrain the bi-pole structure in the x-direction only, as shown in figure 8. The magnitude of the load is measured by a force sensor module (Arduino and beam with a strain gauge as seen in figure 9). The measurement sampling rate is set to 100 Hz s^{-1}. The displacement is incrementally applied with a precise displacement stage. In our experiments, we noticed that the stability of the structure in the horizontal direction is important as the veritical movement is only considered in the snap-through structure. If the BNS mechanism loses the horizontal stability, a different force and displacement curve is obtained. For the sake of this, the guide mechanism composed with several bars and a sliding mechanism shown in figure 10(c) is installed.

Figure 11 shows the force and displacement curves of the present bi-directional mechanism. To obtain the experiment curve in figure 11(b), the force and displacement values are measured 10 times and averaged. With the linear stage used for figures 5 and 11, one full revolution of the knob moves the stage platform 1 mm relative to the stage base. With the linear stage, the mechanism is vertically moved by 0.5 mm and the 1000 force values from the force sensor are averaged. As illustrated, the overall behavior reveals deformations with NS values. With regard to positive displacement, a 12.76 N force is applied around 0 mm. To increase the magnitude of the displacement, the force should be decreased. With regard to a negative displacement, −12.76 N should be applied, with the mechanism revealing a NS structure. There is a jump in the NS structure at 0 mm. This experiment demonstrates the validity of the present mechanism.

4.5. Improved design with a positive spring

In the developed system above, there was a jump in the force and displacement curve at 0 mm. It is our intention to use the above system with a positive or negative preload. However, to remove this jump, it is possible to place positive springs in the locations, as shown in figure 12. Due to our manufacturing limit, it is not tried.

5. Conclusions

This research presented a novel structure exhibiting BNS values with two bi-stable mechanisms and mechanical diodes for mechanical metamaterial applications. Analysis of the dispersion relation of the BNS structure can theoretically apply the bandgap effect to all frequencies. In the present design, a smaller bi-stable mechanism was embedded within a relatively larger bi-stable mechanism in the opposite direction; the subject of the bi-stable mechanism has been studied and it is not a new subject. To filter the applied force to either the small or large bi-stable mechanism, mechanical diodes utilizing mechanical contacts were also embedded within the mechanism. Thus, four mechanical diode structures played an important role with regard to achieving the NS property. To demonstrate the validity of the BNS structure, a mockup structure with guide bars was designed and fabricated from polystyrene. A force sensor and displacement station were employed; the experiment verified the validity of the BNS structure. In conclusion, a BNS stiffness structure was presented. With this novel design, it was possible to possess additional flexibility with regard to the design of new broadband metamaterials, paving the way toward broadband and low frequency insulation materials.

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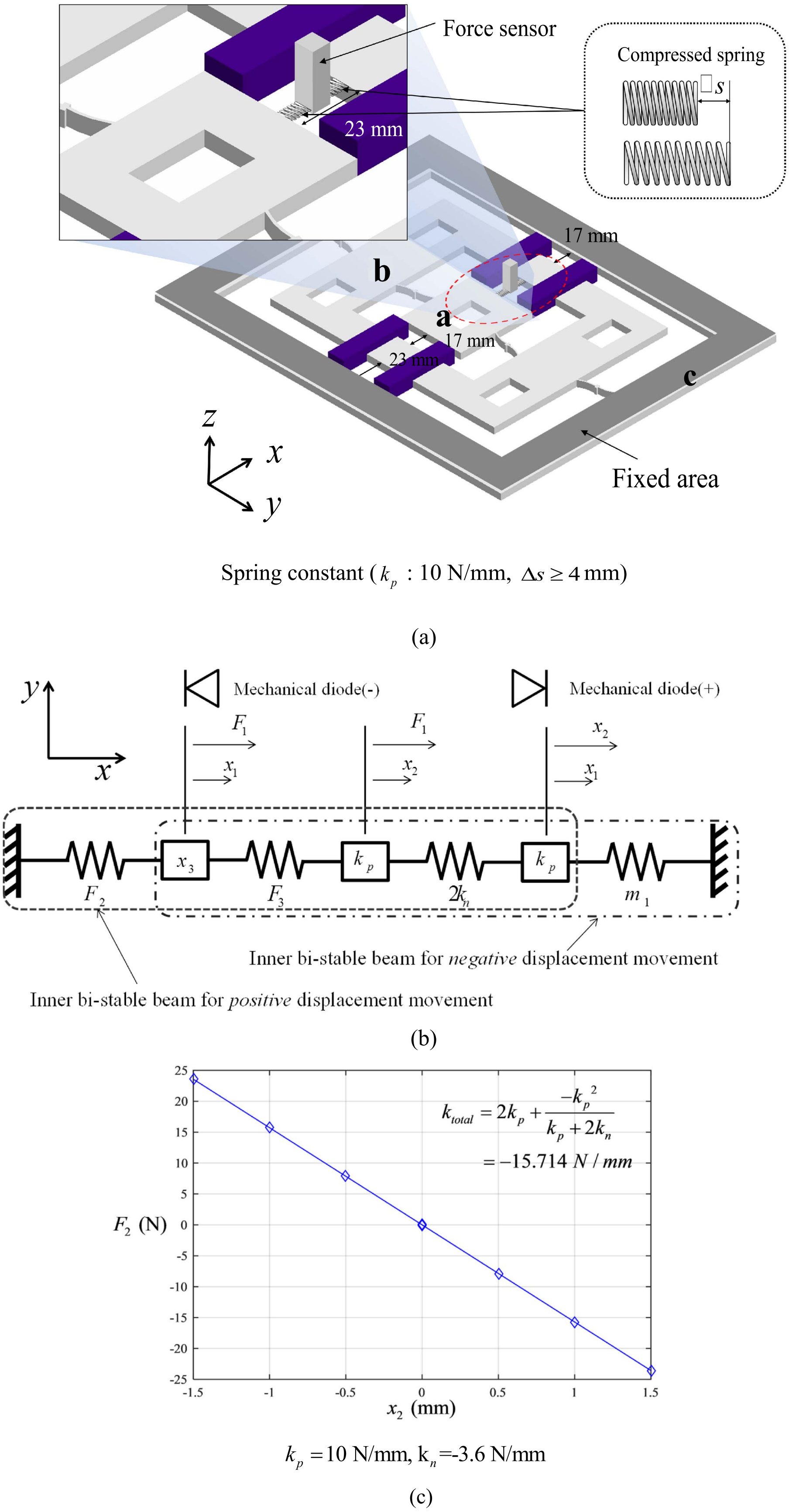

Figure 12. Force and displacement curve of the present bi-directional mechanism with elastic springs.

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References

Awtar S, Shimotsu K and Sen S 2010 Elastic averaging in flexure mechanisms: a three-beam parallelogram flexure case study *Trans. ASME, J. Mech. Robot.* **2** 041006

Awtar S, Slocum A H and Sevincer E 2007 Characteristics of beam-based flexure modules *J. Mech. Des.* **129** 625–39

Chen G and Zhang S 2011 Fully-compliant statically-balanced mechanisms without prestressing assembly: concepts and case studies *Mech. Sci.* **2** 169–74

Chen G M and Ma F L 2015 Kinetostatic modeling of fully compliant bistable mechanisms using Timoshenko beam constraint model *J. Mech. Des.* **137** 022301

Fang N, Xi D J, Xu J Y, Ambati M, SrituraVanich W, Sun C and Zhang X 2006 Ultrasonic metamaterials with negative modulus *Nat. Mater.* **5** 452–6

Florijn B, Coulais C and Van Hecke M 2014 Programmable mechanical metamaterials *Phys. Rev. Lett.* **113** 17–27

Fulcher B A, Shahan D W, Haberman M R, Seepersad C C and Wilson P S 2014 Analytical and experimental investigation of buckled beams as negative stiffness elements for passive vibration and shock isolation systems *Trans. ASME, J. Vib. Acoust.* **136** 031009

Hansen B J, Carron C J, Jensen B D, Hawkins A R and Schultz S M 2007 Plastic latching accelerometer based on bistable compliant mechanisms *Smart Mater. Struct.* **16** 1967–72

Howell L L 2001 *Compliant Mechanisms* (New York: Wiley)

Hutchinson J R 2001 Closure to 'on shear coefficients for Timoshenko beam theory' (2001, ASME J. Appl. Mech., 68 p 959) *Trans. ASME, J. Appl. Mech.* **68** 959

Hwang I H, Shim Y S and Lee J H 2003 Modeling and experimental characterization of the chevron-type bi-stable microactuator *J. Micromech. Microeng.* **13** 948–54

Jensen B D and Howell L L 2003 Identification of compliant pseudo-rigid-body four-link mechanism configurations resulting in bistable behavior *J. Mech. Des.* **125** 701–8

Jensen B D and Howell L L 2004 Bistable configurations of compliant mechanisms modeled using four links and translational joints *J. Mech. Des.* **126** 657–66

Jensen B D, Howell L L and Salmon L G 1999 Design of two-link, in-plane, bistable compliant micro-mechanisms *J. Mech. Des.* **121** 416–23

Jutte C V and Kota S 2008 Design of nonlinear springs for prescribed load–displacement functions *J. Mech. Des.* **130** 081403

Klatt T and Haberman M R 2013 A nonlinear negative stiffness metamaterial unit cell and small-on-large multiscale material model *J. Appl. Phys.* **114** 033503

Lakes R S and Drugan W J 2002 Dramatically stiffer elastic composite materials due to a negative stiffness phase? *J. Mech. Phys. Solids* **50** 979–1009

Lakes R S, Lee T, Bersie A and Wang Y C 2001 Extreme damping in composite materials with negative-stiffness inclusions *Nature* **410** 565–7

Le T D and Ahn K K 2011 A vibration isolation system in low frequency excitation region using negative stiffness structure for vehicle seat *J. Sound Vib.* **330** 6311–35

Lee C M, Goverdovskiy V N and Temnikov A I 2007 Design of springs with 'negative' stiffness to improve vehicle driver vibration isolation *J. Sound Vib.* **302** 865–74

Masters N D and Howell L L 2003 A self-retracting fully compliant bistable micromechanism *J. Microelectromech. Syst.* **12** 273–80

Nicolaou Z G and Motter A E 2012 Mechanical metamaterials with negative compressibility transitions *Nat. Mater.* **11** 608–13

Pucheta M A and Cardona A 2010 Design of bistable compliant mechanisms using precision-position and rigid-body replacement methods *Mech. Mach. Theory* **45** 304–26

Qiu J, Lang J H and Slocum A H 2004 A curved-beam bistable mechanism *J. Microelectromech. Syst.* **13** 137–46

Sonmez U and Tutum C C 2008 A compliant bistable mechanism design incorporating elastica buckling beam theory and pseudo-rigid-body model *J. Mech. Des.* **130** 042304

Timoshenko S 1976 *Strength of Materials* (Huntington, NY: Krieger)

Wang Y C and Lakes R S 2004 Exreme stiffness systems due to negative stiffness elements *Am. J. Phys.* **72** 40–50

Wilcox D L and Howell L L 2005 Fully compliant tensural bistable micromechanisms (FTBM) *J. Microelectromech. Syst.* **14** 1223–35

Zhang A M and Chen G M 2013 A comprehensive elliptic integral solution to the large deflection problems of thin beams in compliant mechanisms *Trans. ASME, J. Mech. Robot.* **5**

Zheng X Y *et al* 2014 Ultralight, ultrastiff mechanical metamaterials *Science* **344** 1373–7