ADVANCED ENGINEERING MATERIALS
www.aem-journal.com

Functionally Graded Phononic Crystals with Broadband Gap for Controlling Shear Wave Propagation

Wenlong Liu, Bing Yi, Gil Ho Yoon,* and Hyunggyu Choi*

Metamaterials that can be used in manipulating wave propagation have been shown in previous research. However, existing methods for controlling the propagation of shear waves remain a challenge. By combining the principle of wave destructive interference and the design concept of the gradient-index phononic crystals, here new functionally graded phononic crystals with broadband gap for controlling shear wave propagation are presented. The proposed functionally graded phononic crystals are formed by an array of unit cells with different topological geometries, where the topological geometries of the unit cell are tailored to obtain the frequency bandgap guided by the wave destructive interference. Meanwhile, the frequency bandgap with a target width is obtained by combining the design concept of the gradient-index phononic crystals. This work presents an approach to control the propagation of shear waves, and the advantages of the method reported in this work can be useful in engineering applications, such as bridges, railways, and buildings.

1. Introduction

Metamaterials, artificial composite materials comprising periodic structures or resonators, have received considerable attention for their qualities of manipulating mechanical wave propagation (e.g., acoustic and elastic waves).[1,2] In the prior few years, a substantial amount of theoretical, numerical, and experimental studies have been conducted on metamaterials with peculiar applications, for example, wave guidance and filtering,[3–5] acoustic,[6–8] energy harvesting,[9–11] and vibration isolation.[12–14] Accordingly, many research groups committed to finding effective methods to realize wave propagation along a predetermined pathway using metamaterials, for example, local resonance methods,[15,16] Bragg scattering methods,[17,18] topology optimization methods,[19–21] and wave interference–based methods.[22] However, to the best of the authors' knowledge, although the existing methods for manipulating wave propagation have been widely investigated, most of them are limited to acting on longitudinal waves and little research work has focused on the shear waves.

To address the aforementioned issues mentioned, one can find that research on shear waves has escalated in recent years. In this regard, Chang et al. proposed a separation method for longitudinal and shear waves propagating simultaneously in a solid medium.[23] Qian et al. explored a method for manipulating shear wave propagation through longitudinal vibration.[24] Kim and his research team successively proposed research methods for shear wave separation and mode-converting transmission.[25–27] The reported gradient-index phononic crystals (GRIN PnCs) for energy harvesting are based on shear-vertical mode waves and A0 mode Lamb waves.[28,29] Despite the existing research works providing some ways to manipulate shear wave propagation to some extent, controlling shear wave propagation remains an open challenge in designing structures with the desired bandgaps.

Herein, we explore and demonstrate new functionally graded phononic crystals that can effectively control the propagation of shear waves. Our work is based on wave destructive interference, which has been applied in the fields of acoustics, optics, among others, for example in acoustic absorption,[22,30] optical systems,[31–34] and precision measurement.[35] It should be mentioned that, although wave destructive interference can control the propagation of shear waves, the bandgaps produced by this theory are limited to a narrow frequency range, which is still restricted in practical applications. For this reason, by combining with wave destructive interference, we introduce the design concept of GRIN PnCs to overcome the shortcomings of bandgap structure with a narrow frequency range. A similar method using gradient design to overcome narrow bandgaps can be found in Wang et al. and Sun et al.[34,36] However, with the gradient index, unlike the previous GRIN PnCs, we extend the original concept of gradient refractive index to the functionally graded structure, that is, the gradient arrangement of unit cells with different bandgaps. The remainder of this article is organized as follows. In Section 2, we detail the mechanism of wave destruction interference in structural design and realize unit cells with different bandgaps, and then periodically arrange them to obtain the functionally graded phononic crystals. In Section 3, we demonstrate numerically and experimentally that the functionally graded phononic crystals can effectively control the propagation

Dr. W. Liu, Prof. B. Yi
School of Traffic and Transportation Engineering
Central South University
Hunan 410075, P. R. China
E-mail: bingyi@csu.edu.cn

Prof. G. H. Yoon, Dr. H. Choi
School of Mechanical Engineering
Hanyang University
Seoul 04763, South Korea
E-mail: ghy@hanyang.ac.kr

The ORCID identification number(s) for the author(s) of this article can be found under https://doi.org/10.1002/adem.202000645.

DOI: 10.1002/adem.202000645

of shear waves. Finally, the conclusion is presented in Section 4. The experimental details are provided in the Supporting Information.

2. Functionally Graded Phononic Crystals

In this section, first, we describe the application of the physical principle of wave destructive interference in functionally graded phononic crystals. As shown in **Figure 1**a, the wave destructive interference is given, and on the basis, we use a simple unit structure to describe the mechanism of bandgap formation in functionally graded phononic crystals, as shown in Figure 1b, and the bandgap expression corresponding to the structure is defined as

$$\delta = \ell_1 - \ell_2 = (k + 1/2)\lambda, \; k = 0, 1, 2, \cdots \tag{1}$$

where ℓ_1 and ℓ_2 are long and short paths of wave propagation and λ is the wavelength, which satisfies the relationship $c_s = \lambda \cdot f$. Here, c_s is the wave speed propagating in the medium; as we already we know, the wave velocity c_s mainly depends on the medium and can be calculated using $c_s = \sqrt{E/2(1+\mu)\rho}$, where E, μ, ρ are the elastic modulus, Poisson's ratio, and density of the medium. Based on the wave destructive interference described in Equation (1), as an example, we design a unit cell with $\ell_1 = 0.5371$ m, $\ell_2 = 0.5$ m, and $k = 0$, as shown in Figure 1b. The stress value and the bandgap structure of the unit cell are shown in Figure 1c,d (for details please see the Supporting Information S1). In addition, some special destructive interference snapshots of the corresponding waves with different wavelengths when $k \neq 0$ are shown in Figure 1e, and it is this situation that provides the possibility of controlling wave propagation in the low-frequency range (details can be seen in Figure S2, Supporting Information).

According to the aforementioned theoretical analysis and numerical simulation results, in the following, we design a gradient unit structure with a gradient bandgap, as shown in Figure 1f (the numerical results of the functionally graded structure corresponding to each unit structure can be found in Supporting Information S3). Then, we arranged the different gradient unit structures to obtain functionally graded phononic crystals, which are used to control the propagation of shear waves, as shown in Figure 1g. Note that the design of the functionally graded phononic crystals needs to be based on Equation (1) and related parameters therein, as well as the target frequency bandgap. In this work, the material parameters of the functionally graded phononic crystals are as follows: ρ=2700 kg m^{-3}, $E = 70$ GPa, $n_u = 0.33$, and the geometric parameters $l \times w \times h = 50\,\text{cm} \times 10\,\text{cm} \times 0.5\,\text{cm}$. The central frequency of the designed functionally graded phononic crystals is selected as $f = [900, 1200, 1600, 2000]$Hz.

3. Results and Discussion

3.1. Numerical Analysis

To investigate the effect of the functionally graded phononic crystals on controlling the propagation of shear waves, finite element analysis was first used to develop relevant analysis. It should be mentioned that throughout this article, the finite element analysis is performed in the frequency domain of solid structure interfaces in Comsol Multiphysics. In the numerical simulation, two ends of the functionally graded phononic crystals are free; an external force is applied at one end and the frequency response measured at the other end in the y-direction. The bandgap structures of the functionally graded phononic crystals are obtained using $T = 20lg(A_c/A_r)$, where A_c is the acceleration value at the measuring point and A_r is a reference value; usually, $A_r = 10e^{-5}\,\text{m s}^{-1}$. **Figure 2** and **3** show the numerical simulation

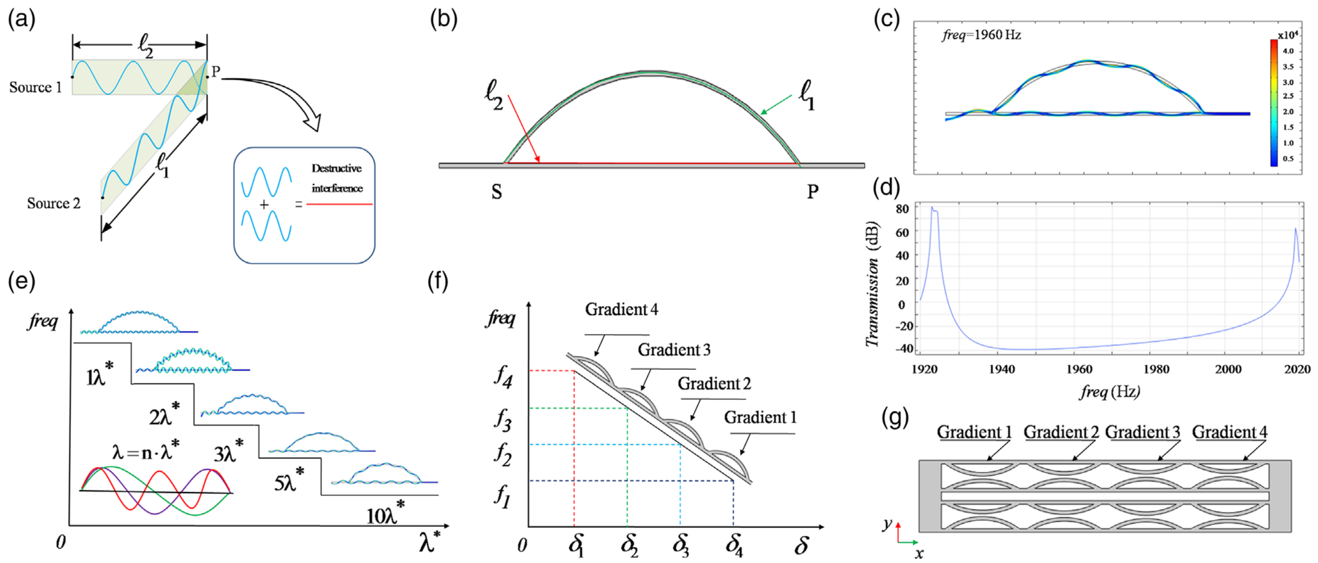

Figure 1. Schematic diagram of the mechanism of functionally graded phononic crystals: a) destructive interference, b) a unit cell, c) the stress value of a unit cell, d) the bandgap structure of a unit cell, e) numerical simulation of destructive interference of transverse waves with different wavelengths, f) gradient design of the unit cells with different frequency bandgaps, and g) schematic diagram of functionally graded phononic crystals.

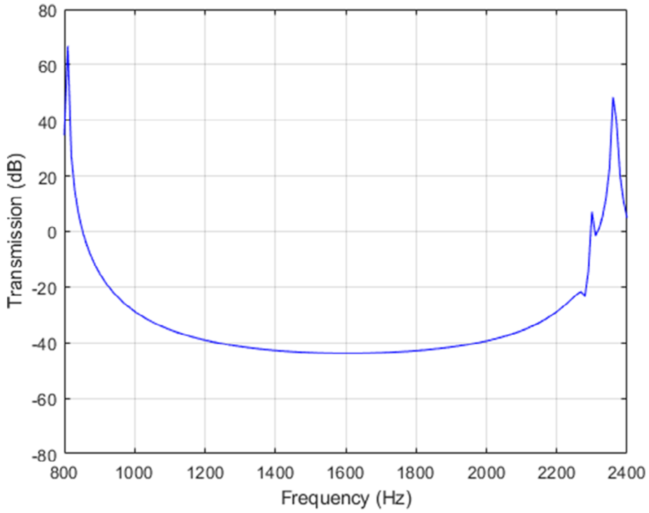

Figure 2. The bandgap structure of functionally graded phononic crystals.

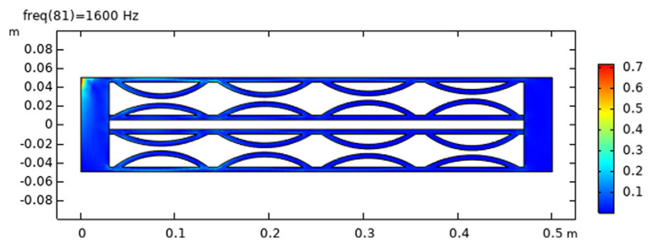

Figure 3. Dynamic result of numerical simulation (Supporting Information S4).

results of the functionally graded phononic crystals in controlling the propagation of shear waves. Specifically, Figure 2 shows the bandgap structure of the functionally graded phononic crystals, which is consistent with the designed frequency bandgap structure range. Figure 3 shows the details of the shear wave propagation being controlled using the dynamic result of the numerical simulation (details in Figure S8, Supporting Information). The two aforementioned numerical simulation results verify the capability of functionally graded phononic crystals to control the propagation of shear waves.

3.2. Experiments

To further validate our design, frequency response measurements were performed on the fabricated functionally graded phononic crystals (which were fabricated by a laser cutting technique, as shown in **Figure 4**) based on our experimental setup.

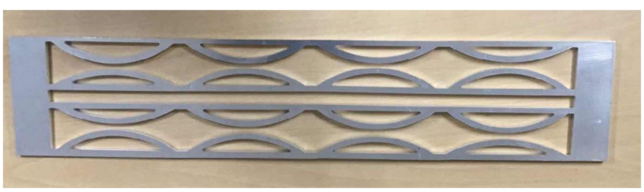

Figure 4. The fabricated functionally graded phononic crystals.

To ensure the effective comparison between experimental results and numerical simulation results, it is worth remarking that the conditions set in this experiment should be coherent with those in the numerical simulation, which consist of the degree of freedom of the functionally graded phononic crystals, the position of the impact force, and the location and direction of acceleration device. In this experiment, the experimental setup included data acquisition (NI9234), an impact force hammer, an accelerator, and a computer; the specific settings are shown in **Figure 5**. Comparison between the simulated and experimentally measured bandgap structure of the fabricated functionally graded phononic crystals is shown in **Figure 6**. As shown in Figure 6, the experimentally measured frequency bandgap range is consistent with the numerical simulation results in Figure 2, except for some details, such as the fluctuation amplitude, which were caused by the manufacturing and experimental error. However, the most important point that one cannot neglect is that both the aforementioned results demonstrate that the propagation of the shear waves can be controlled in the target frequency bandgap range by utilizing the functionally graded phononic crystals.

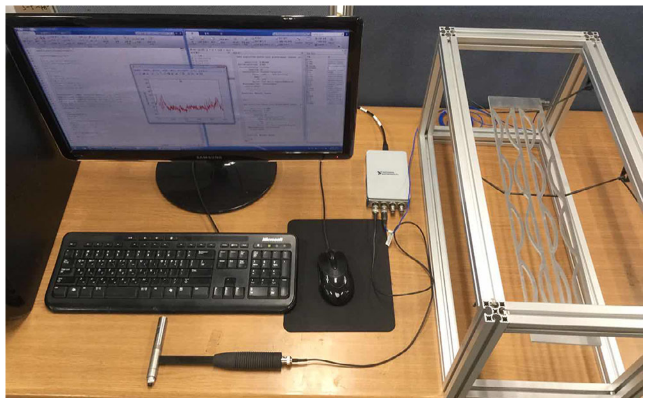

Figure 5. Experimental setup.

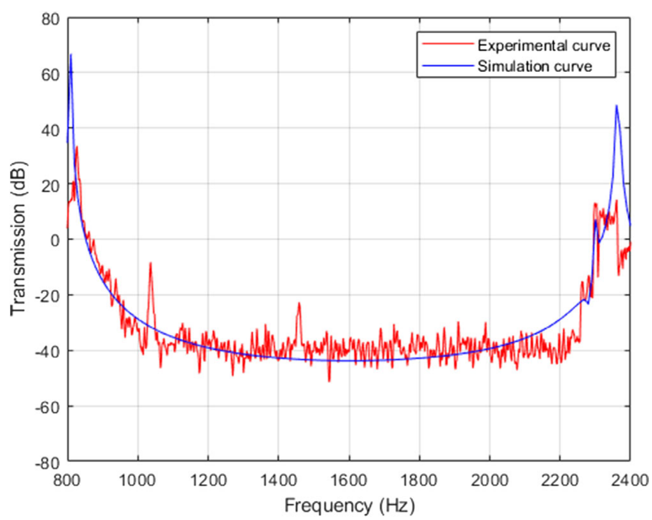

Figure 6. Simulated and experimentally measured bandgap structure of functionally graded phononic crystals.

4. Conclusion

In this work, we designed, fabricated, and experimentally validated functionally graded phononic crystals composed of different unit cells with functionally graded structures that can effectively control the propagation of shear waves. It demonstrates that the combination of GRIN PnCs and wave destructive interference theory can make up for the shortcomings of narrow bandgap produced by wave interference theory alone, and the design of GRIN PnCs can be used to flexibly design functionally graded phononic crystals with a broadband gap. It should be emphasized that the functionally graded phononic crystals in this work are only one kind of structure that meets the proposed method, and the readers can design other functionally graded phononic crystals with the target frequency based on the proposed method, for example, the other two functionally graded phononic crystals in Supporting Information S5. In conclusion, our work provides a way to control the propagation of shear waves, and the advantages of the method described in this work can be useful in engineering applications, such as bridges, railways, and buildings.

Future research to build upon this work will explore tunable functionally graded phononic crystals in the desired frequency range, which is one of the development trends of metamaterials designs, as described in Nemati et al..[37] In addition, solving the actual manufacturing problems based on the proposed functionally graded phononic crystals is also worthy of further research. For example, the control of the generation of shear waves during laser shock peening, which is mentioned in Wu et al.[38]

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

Acknowledgements

This work was conducted during the first author's visit to Han Yang University. This research was supported by the National Natural Science Foundation of China under Grant No. 51975589 and No. 51605495, Fundamental Research Funds for the Central University of Central South University No. 2019zzts267, and Central South University Graduate Study Abroad Exchange Program No. 31801-160170002. The authors gratefully acknowledge these sources of support.

Conflict of Interest

The authors declare no conflict of interest.

Keywords

broadband gap, functionally graded, phononic crystals, shear waves

Received: May 29, 2020
Revised: July 12, 2020
Published online: July 30, 2020

[1] N. Kaina, M. Fink, G. Lerosey, *Sci. Rep.* **2013**, *3*, 3240.
[2] R. Martínez-Sala, J. Sancho, J. V. Sánchez, V. Gómez, J. Llinares, F. Meseguer, *Nature* **1995**, *378*, 241.
[3] M. Kafesaki, M. M. Sigalas, N. Garcia, *Phys. Rev. Lett.* **2000**, *85*, 4044.
[4] A. Khelif, B. Djafarirouhani, J. O. Vasseur, P. A. Deymier, *Phys. Rev. B* **2003**, *68*, 024302.
[5] a) J. Sun, T. Wu, *Phys. Rev. B* **2006**, *74*, 174305; b) J. Sun, T. Wu, *Phys. Rev. B* **2007**, *76*, 104304.
[6] S. S. Lin, B. R. Tittmann, J. Sun, T. Wu, T. J. Huang, *J. Phys. D* **2009**, *42*, 185502.
[7] T.-T. Wu, Y.-T. Chen, J.-H. Sun, S.-C. S. Lin, T. J. Huang, *Appl. Phys. Lett.* **2011**, *98*, 171911.
[8] Z. Cai, S. Zhao, Z. Huang, Z. Li, M. Su, Z. Zhang, Z. Zhao, X. Hu, Y. Wang, Y. Song, *Adv. Funct. Mater.* **2019**, *29*, 1906984.
[9] H. Lv, X. Tian, M. Y. Wang, D. Li, *Appl. Phys. Lett.* **2013**, *102*, 034103.
[10] S. Gonella, A. C. To, W. K. Liu, *J. Mech. Phys. Solids* **2009**, *57*, 621.
[11] X. Liang, T. Jiang, G. Liu, T. Xiao, L. Xu, W. Li, F. Xi, C. Zhang, Z. L. Wang, *Adv. Funct. Mater.* **2019**, *29*, 1807241.
[12] A. S. Phani, J. Woodhouse, N. A. Fleck, *J. Acoustical Soc. Am.* **2006**, *119*, 1995.
[13] M. Zheng, P.-J. Wei, *Int. J. Miner. Metall. Mater.* **2009**, *16*, 608.
[14] K. H. Matlack, A. Bauhofer, S. Krödel, A. Palermo, C. Daraio, *Proc. Natl. Acad. Sci.* **2016**, *113*, 8386.
[15] S. Li, Y. Dou, T. Chen, J. Xu, B. Li, F. Zhang, *Phys. Lett. A* **2019**, *383*, 1371.
[16] W. Jihong, W. Gang, Y. Dianlong, Z. Honggang, L. Yaozong, W. Xisen, *Sci. China-Technol. Sci.* **2008**, *51*, 85.
[17] C. Kao, S. Osher, E. Yablonovitch, *Appl. Phys. B* **2005**, *81*, 235.
[18] Y. Wang, F. Li, W. Huang, Y. Wang, *J. Phys.: Condens. Matter* **2007**, *19*, 496204.
[19] a) H. Dong, X. Su, Y. Wang, C. Zhang, *Struct. Multidisc. Optim.* **2014**, *50*, 593; b) H. Dong, X. Su, Y. Wang, C. Zhang, *Phys. Lett. A* **2014**, *378*, 434.
[20] X. Yang, J. S. Lee, Y. Y. Kim, *J. Sound Vibr.* **2016**, *383*, 89.
[21] G. Yi, Y. C. Shin, H. Yoon, S.-H. Jo, B. D. Youn, *JMST Adv.* **2019**, *1*, 153.
[22] J. Sun, L. Liu, G. Dong, J. Zhou, *Opt. Express* **2011**, *19*, 21155.
[23] Z. Chang, H. Guo, B. Li, X. Feng, *Appl. Phys. Lett.* **2015**, *106*, 161903.
[24] Y. Qian, Q. Cui, X. Yang, W. Zhang, *Int. J. Mech. Sci.* **2020**, *168*, 105296.
[25] X. Yang, Y. Y. Kim, *Phys. Rev. B* **2018**, *98*, 144110.
[26] C. I. Park, H. M. Seung, Y. Y. Kim, *Ultrasonics* **2019**, *99*, 105958.
[27] M. Zheng, C. I. Park, X. Liu, R. Zhu, G. Hu, Y. Y. Kim, *Appl. Phys. Lett.* **2020**, *116*, 171903.
[28] a) S. Tol, F. L. Degertekin, A. Erturk, *Appl. Phys. Lett.* **2016**, *109*, 063902; b) S. Tol, F. L. Degertekin, A. Erturk, *Appl. Phys. Lett.* **2017**, *111*, 013503; c) S. Tol, F. L. Degertekin, A. Erturk, *Addit. Manuf.* **2019**, *29*, 100780.
[29] J. Hyun, W. Choi, M. Kim, *Appl. Phys. Lett.* **2019**, *115*, 173901.
[30] A. Arjunan, *Mater. Today Commun.* **2019**, *19*, 68.
[31] W. Guo, *Am. J. Phys.* **2006**, *74*, 595.
[32] L. Chen, Y. Yin, Y. Li, M. Hong, *Opto-Electron. Adv.* **2019**, *2*, 190019.
[33] H. A. Macleod, *Thin Film Optical Filters*, 2nd ed., Adam Hilger Ltd., Bristol, UK **1986**.
[34] X. Wang, Y. Wang, J. Flueckiger, R. Bojko, A. W. K. Liu, A. Reid, J. Pond, N. A. F. Jaeger, L. Chrostowski, *Opt. Lett.* **2014**, *39*, 5519.
[35] X. Li, J. Gao, L. Xue, Y. Han, *Adv. Funct. Mater.* **2010**, *20*, 259.
[36] H. Sun, W. Tian, X. Wang, K. Deng, J. Xiong, L. Li, *Adv. Mater.* **2020**, *32*, 1908108.
[37] A. Nemati, Q. Wang, M. Hong, J. Teng, *Opto-Electron. Adv.* **2018**, *1*, 180009.
[38] J. Wu, J. Zhao, H. Qiao, X. Liu, Y. Zhang, *Opto-Electron. Adv.* **2018**, *1*, 180016.

