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Controlling wave propagation in one-dimensional structures through topology optimization

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ABSTRACT

Research on the control of wave propagation has received continuous attention due to its potentially rewarding applications in the past decades, and numerous methods have been developed for controlling wave propagation in certain materials or structures. Despite previous work has made many innovations in controlling wave propagation, they are limited to the research from a band gap perspective. Herein, this paper presents a gradient-based multi-functional topology optimization for controlling wave propagation in a one-dimensional (1D) structure, which can realize the control of wave propagation from two aspects: band gap and wave propagation speed. To illustrate the method, three case studies are investigated to obtain the following: (1) increasing the band gap width, (2) controlling the wave propagation at target speed, and (3) limiting the propagation of low-frequency waves. By evaluating the results of three case studies, the effectiveness of the proposed topology optimization method is demonstrated. More importantly, the control of wave propagation in the low-frequency range in Case III lends new insight into the vibration isolation structure in engineering applications.

1. Introduction

Vibration is a common phenomenon in nature, and its manifestation is the propagation of waves in the structure. However, some vibrations in the structure are undesirable because it may produce unwanted results, such as vibration damage and sound noise. Therefore, the development of effective vibration isolation methods has become an urgent problem in various engineering applications and has been studied for decades.

Over the past decades, the control of wave propagation in periodic engineering structures has always been the research hotspot [1–4]. In this regard, some typical examples of controlling wave propagation can be found, to name a few, modulated rods [5,6], supported beams [7,8], lattice structures [9,10], plates [11,12], etc. In addition, phononic crystals (PCs) have made it more helpful to control the propagation of waves in periodic structures. What's special is that the wave filtering property (also called band gap) exists in PCs, which makes it possible to prohibit the mechanical waves (i.e., elastic and acoustic waves) from propagating in a certain frequency range. Such a unique property has been studied in many engineering applications in recent years, such as wave splitting [13], wave guidance and filtering [14–16], and sound or vibration isolation [17–19], etc.

However, after an in-depth analysis of the current methods for controlling wave propagation in existing literature, we find that previous work limited to the research from the perspective of the band gap, and these methods are to find the band gap and then increase its width. Furthermore, to the best of our knowledge, these research methods for band gap are focusing on two types of methods: passive methods and topology optimization, yet these two categories of methods have shortcomings to some extent, which are discussed in Section 2.

In this context, this paper presents a gradient-based multi-functional topology optimization for controlling wave propagation in periodic structures through topology optimization. As a simple illustration example, 1D structure was studied in this paper. The specialty of this proposed method is that it can realize the control of wave propagation from two aspects: band gap and wave propagation speed. In particular, the proposed method for controlling wave propagation speed in a periodic structure through topology optimization has not been published yet. The importance and originality of this research are it explores a new way to control the propagation of waves in periodic structures that can be used in the high, medium and low-frequency ranges. Meanwhile, this method can realize the important contribution of the wave propagation controlling in the desired target speed. It should be empha-

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sized that, as a special case where the target speed is zero, this method also provides an effective research point for limiting the propagation of the waves.

The remainder of this paper is organized as follows. Section 2 discusses the review of related literature. The theory of wave propagation in periodic structures and the settings for propagation in a 1D structure are described in Section 3. The multi-functional topology optimization for controlling wave propagation in the 1D structure is given in Section 4. Three case studies and experiments are presented in Section 5. Finally, the conclusion and future work are presented in Section 6.

2. Literature review

2.1. Passive methods

To control the wave propagation in a periodic structure, extensive efforts have been exerted and considerable methods have been developed. Among them, the passive methods (also called reactive methods) are the earliest methods, which use the discontinuity design of geometric or material (i.e., impedance mismatch zones) in a periodic structure to impede and attenuate the propagation of waves from one end of the structure to the other, as shown in Fig. 1(a) and (b) [20]. The proposed impedance mismatch zone in the passive method has a very profound impact, manifested in two aspects: (1) a periodic structure in an impedance mismatch region that allows waves to pass or stop in a selected frequency band (also known as the pass or stop band gaps), and (2) the width of these band gaps in periodic structure can be controlled. Many scholars have done a lot of research work, which can be found in [21] and references therein. Based on these research conclusions mentioned above, a similar method for controlling wave propagation using stop band was developed in later research, local resonance and the schematic diagram as shown in Fig. 1(c). Overall, these methods use negative effective indices (e.g., mass density, bulk, and shear modulus) to create band gaps, thereby controlling the wave propagation in a periodic structure, which can also be referred to as passive methods.

Based on the methods described above, several methods and experiments are performed for controlling wave propagation in periodic structures. For instance, Mead and his coworkers studied the width and location of band gaps to control the wave propagation, excellent works can be found in [3,22] and references therein. Joo et al. [23] proposed a method to control wave propagation along a designated direction by adjusting the negative density and stiffness. Sang and Wang [24] explored the integration of periodic honeycomb lattices and inclusions to get periodic material for controlling low-frequency wave propagation. Yao et al. [25,26] used a mass-spring system to explain the local resonance transmission characteristics of the basic unit in the low-frequency range through experiments. While these efforts have shown that passive methods can create band gaps to control wave propagation, it takes a lot of time to trial-and-error to obtain the impedance mismatch zones in the desired frequency range, such as adjusting the geometry, orientation, position, density, and spring-mass parameters.

2.2. Topology optimization

The first-reported work on topology optimization by Bendsoe and Kikuchi is to solve structural optimization of mechanical design problems [27]. In recent years, this method has been widely used to solve the optimization of material distribution, which can be found in [28–30]. Specifically, the research on band gap properties of PCs is active. To illustrate, the earliest work performed by Sigmund and Jensen [31], they introduced a solid isotropic material with penalization (SIMP) method to achieve the optimization of material distribution to maximize the band gap width. Kao et al. [32] introduced a level set method to maximize band gaps in two-dimensional PCs, and the effects of proposed methods of band gap optimization under different design regions are studied, such as rotation, mirror-reflection, and inversion symmetry. Dong et al. [33] combined the finite element method and a genetic algorithm to optimize the band gap width of two-dimensional PCs. Li et al. [34] explored a topology optimization algorithm based on bidirectional evolutionary structural optimization method and finite element analysis for the design of band gap in 2D PCs. Yang et al. [35] proposed a topology optimization method that expands the range of negative effective mass density to obtain a material design with 2D band gap maximization. Despite these methods can increase the band gap width, they are all initialized by using a regular-shaped inclusion method, as shown in Fig. 1(d). This initial setting of inclusion shape may not result in optimal design for the maximum band gap width.

In parallel with the regular-shaped inclusion method described above, some researchers use topology optimization for a single-material to obtain the maximum band gap width for controlling wave propagation. For example, Dong et al. [36] used an adaptive algorithm and genetic algorithm to obtain the maximum band gap of 2D asymmetric PCs. Li et al. [37] explored the widest band gap of cellular PCs using the bidirectional evolutionary structure optimization algorithm and the homogenization method, and obtain the optimized topologies and corresponding maximum band gap under different constraints. Halkjæret al. [38] investigated the maximum band gap of Mindlin plate structure for bending waves by using the topology optimization method. While these methods can get the maximum band gap, over-reliance on the

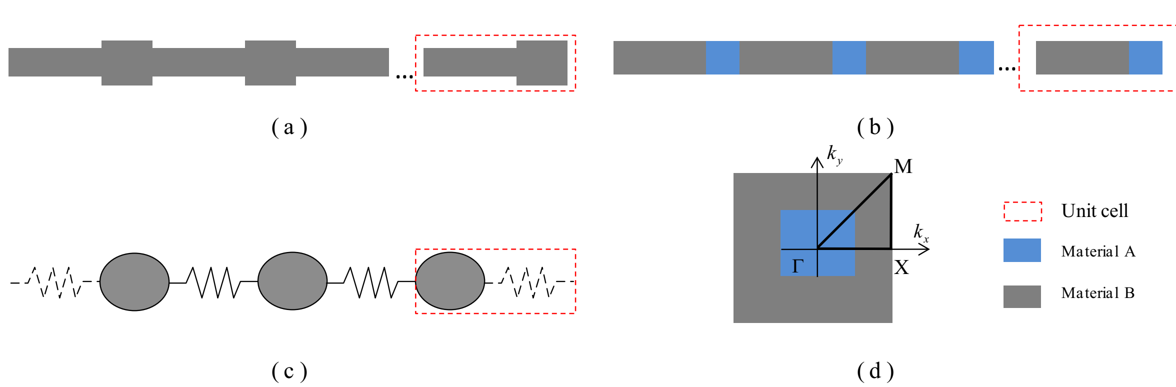


Fig. 1. Periodic structure (a) discontinuity design of geometric; (b) discontinuity design of material; (c) one period of infinite periodic mass-spring structure; (d) unit cell comprising regular inclusions.

existing band gap in a periodic structure limits the practical application to some extent.

3. Wave propagation in periodic structures

The governing equation to the elastic waves can be expressed as follows.

$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial^2 t} = (\lambda + 2\mu)\nabla\nabla \cdot \boldsymbol{u} - \mu\nabla \times \nabla \times \boldsymbol{u} \tag{1}$$

where λ and μ are Lamè's coefficients, ρ is the material density of solid, t is time and $\boldsymbol{u}$ is a vector of the displacement fields.

Assuming that the wave propagation to be confined to the x-y plane only and the unit cell is periodically arranged, as shown in Fig. 2(a), the periodic boundary conditions [39] are applied to the interfaces adjacent to the other unit cells according to the Bloch-Floquet theory [40], as follows:

$$\mathbf{u}\{A_1, B_1, t\} = \mathbf{u}\{A_2, B_2, t\}e^{i(k_x b + k_y h)} \tag{2}$$

where k_x and k_y are components of a reciprocal wave vector in the x- and y-directions, respectively. By substituting Eq. (2) into Eq. (1), the eigenvalue problem can be formulated as follows:

$$([\mathbf{K}(k_x, k_y)] - \omega^2[\mathbf{M}])\{\mathbf{u}\} = 0 \tag{3}$$

where $\mathbf{K}$ and $\boldsymbol{M}$ are the stiffness matrix and mass matrix.

Based on the above analysis, the problem of wave propagation in solids can be transformed into a problem of solving eigenvalues, and the dispersion curve of a unit cell can be numerically obtained by calculating eigen-frequencies for a given reciprocal wave vector, which is swept along the borderline of irreducible Brillouin zone (IBZ) (as shown in Fig. 1(d)). For example, the parameter combination of $k_x = 0$ and $k_y = [-\pi, \pi]$ can be used to analyze the eigen-frequencies in XM direction of the IBZ, and $k_x = k_y \in [-\pi, \pi]$is for the ΓM direction. As band diagrams around $k_i = 0 (i = 1, 2)$are symmetric, in this study, the setting corresponding to the longitudinal wave (along ΓX direction) in this research is the parameter combination of $k_y = 0$ and $k_x = [0, \pi/\mathrm{b}]$.

4. The multi-functional topology optimization

In this research, a multi-functional topology optimization for controlling wave propagation in periodic structures is proposed, which can realize the control of wave propagation from two aspects: band gap width and wave propagation speed. This method follows the SIMP based topology optimization method proposed in [41] for material distribution. The mathematical formulation of this topology optimization problem can be defined as.

$$\begin{aligned} &\max \text{or} \ \min: \ \mathrm{f}(\Sigma, \gamma) \\ &\text{subjectto}: \ \mathrm{V}(\gamma)/\mathrm{V}_0 = \mathrm{f} \\ &[\mathbf{K}^*(\boldsymbol{k}) - \omega^2\mathbf{M}^*(\boldsymbol{k})]\{\mathbf{u}_M\} = 0 \\ &\gamma_{\min} < \gamma < 1, \ e = 1, 2, \cdots, N \end{aligned} \tag{4}$$

where Σ denotes the topological distribution within the unit cell of the periodic structure. $f(\Sigma, \gamma)$ is the objective function of the topological distribution that aims to be maximized and minimized, which is presented in the next subsection. γ represents the vector of design variables, γ_{min} is the small design variable assigned to void regions to prevent the matrix from becoming singular. N is the number of elements used to discrete the design domain. $V(\gamma)$ and V_0 are the material volume and design domain volume, and f is the prescribed volume fraction. The definition of other parameters is consistent as given in Section 3.

4.1. Optimization formulation

As mentioned above, the proposed topology optimization in this research is multi-functional, which is reflected in the two objective functions (i.e., the band gap width and wave propagation speed), and will be described in detail in the following sections.

4.1.1. Band gap width

When the band gap width is used as the objective function of the optimization problem, the purpose is to control the wave propagation by maximizing the band gap in desired frequency ranges. In this paper, the absolute band gap as the objective function to evaluate the band gap width during the optimization process. The mathematical expression of the absolute band gap width is as follows

$$\max: \ \mathrm{f}(\Sigma, \gamma) = \omega_{\mathrm{j+1}}(\mathbf{k}) - \omega_{\mathrm{j}}(\mathbf{k}) \tag{5}$$

where ω_j is the j-th eigen-frequency. Herein, considering that the absolute band gap in this paper is similar to the existing method to maximize the band gap, it will not be repeated here. The readers can refer to [42] and the references therein.

4.1.2. Wave propagation speed

In the control wave propagation problem, it is a new research point to take the wave propagation speed as the objective function of topology optimization. The topological geometric result of the optimization problem can realize the wave propagation at a target

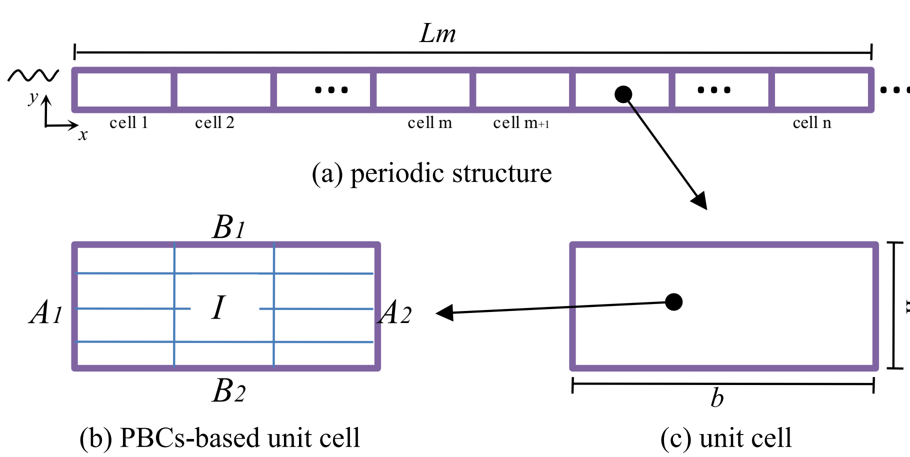


Fig. 2. The waves propagate in a periodic structure.

speed. In the following sections, a detailed description is given for this objective function.

The dispersion curve is an important characteristic when waves of different wavelengths propagate in a dispersive medium. The dispersion relation for the frequency ω and wave vector $\boldsymbol{k}$ can be written as

$$\omega = \omega(\boldsymbol{k}) \tag{6}$$

Additionally, for the j-th point on the dispersion curve, the wave propagation speed c is [28],

$$c = \frac{\omega(k_j)}{k_j} \tag{7}$$

Inspire by Eq. (7), the expression of the slope equation expressed by the two points at the end of the dispersion curve is proposed,

$$\Delta s = \frac{\Delta\omega(k)}{\Delta k} \tag{8}$$

Eq. (8) is the function proposed in this work to control the wave propagation speed, as shown in Fig. 3(a), and then combine it with the target value, the mathematical expression of the objective function in topology optimization problem of controlling the wave propagation speed is obtained, as follows

$$\min:\ f(\Sigma, \gamma) = \left|c_s(k) - c_{target}(k)\right| \tag{9}$$

where c_s is a functional function associated with Eq. (8), and c_{target} denotes the target value designed byresearcher. According to the setting of the target value, the topology optimization problem of controlling the wave propagation speed can be divided into two categories: (1) the target speed is a non-zero constant, which means the wave will propagate in at a constant speed, and (2) the target value is zero, and the optimized result of the slope of the dispersion curve is zero, as shown in Fig. 3(b), this situation is a special case, which indicates that the corresponding wave is no longer propagating in the periodic structure.

Here, what needs a special explanation is that only the slope of the end dispersion curve is used as the slope objective to control the wave propagation speed, as shown in Fig. 3. Therefore, the slope of the dispersion curve mentioned in the following research is the slope of the end dispersion curve. To make it clear, we use the inflection point (Δ) to mark the beginning of the slope of the dispersion curve that tends to be stable, yet the inflection point has no practical significance in solving the topology optimization problem.

4.2. Sensitivity analysis

One of the important components of a topology optimization problem is the sensitivity analysis, which plays an important role in updating of design variables. As described in Section 3, the topology optimization problem related to controlling wave propagation in periodic structures is a problem of solving eigenvalues. Consequently, the derivative of eigen-frequencies associated with problem in Eq. (3) is the sensitivity analysis of topology optimization problem, we have,

$$\frac{\partial\omega_j(\boldsymbol{k})}{\partial\gamma_e} = \frac{1}{2\omega_j}\{\mathbf{u}_j\}^T\left[\frac{\partial\mathbf{K}^*(\boldsymbol{k})}{\partial\gamma_e} - \omega_j^2\frac{\partial\mathbf{M}^*(\boldsymbol{k})}{\partial\gamma_e}\right]\{\mathbf{u}_j\} \tag{10}$$

where $\{\mathbf{u}_j\}$ is the displacement vector (eigenvector) corresponding to the j-th eigen-frequency. $\partial\mathbf{K}^*/\partial\gamma_e$ and $\partial\mathbf{M}^*/\partial\gamma_e$ are the derivatives of the global stiffness and mass matrices. Note that one eigen-frequency corresponds to one eigenvector, and the sensitivity analysis for the objective functions in Eq. (5) and Eq. (9) can be calculated by Eq. (10) for the sensitivity analysis for each element 'density'.

5. Results and discussion

In this section, three case studies are presented and their results are discussed in detail. The material parameters used in the calculations are as follows: ρ = 7850 kg/m^3, E = 200 GPa, n_u = 0.3. The unit cell of the periodic structure$b \times \mathrm{h} = 0.15\ m \times 0.03\ m$, as shown in Fig. 4(b). The meshing size on the unit cell240 $\times$ 48, the volume fraction is 0.7, the penalty factor is 3 and the filtering radius is set as 0.03 times the element width. This topology optimization problem is performed using Matlab and solved using the gradient method (MMA) presented in [41], which is a gradient-based optimizer without a multi-start strategy, thus realizing a single optimization run can only generate a local minimum. The maximum number of iterations is 100, and the convergence accuracy is 0.001.

In addition, to avoid obtaining the result that all waves cannot propagate based on the initial unit cell of the periodic structure, as shown in Fig. 4(b), we used Fig. 4(c) and (d) to conduct the research, where the geometry of Fig. 4(d) is used as a comparison with the analysis results of Fig. 4(c) to further verify the effectiveness of the method. Furthermore, the blue indicates the design domain, yellow indicates the non-design domain. For clarity, the geometry shown in Fig. 4(c) and the research based on it are referred to as G1, and the geometry shown in Fig. 4(d) and the research based on it are referred to as G2.

5.1. Case Ⅰ: Increasing the band gap width

The first case study was to verify that this method can be used to increase the width of the band gap like the existing method. In this research, the position and width of the existing band gap corresponding to the material parameters and design domain in Fig. 4(b), as shown in the shading areas in Fig. 4(a). It can be seen from

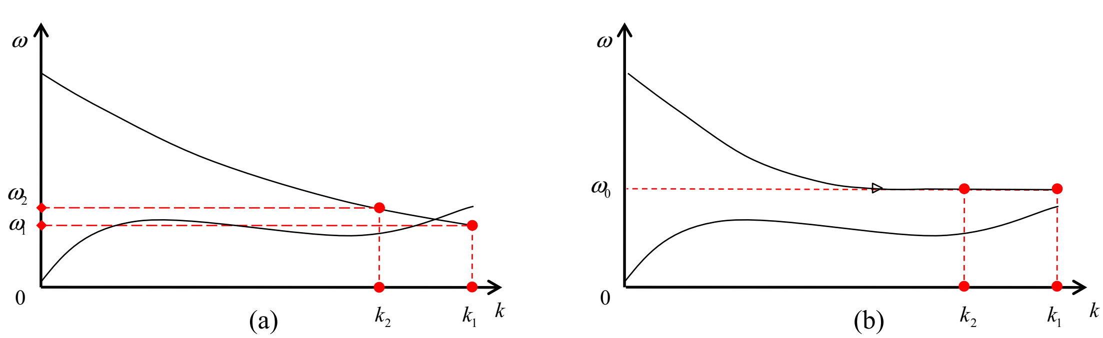


Fig. 3. Slope equation of the dispersion curve.

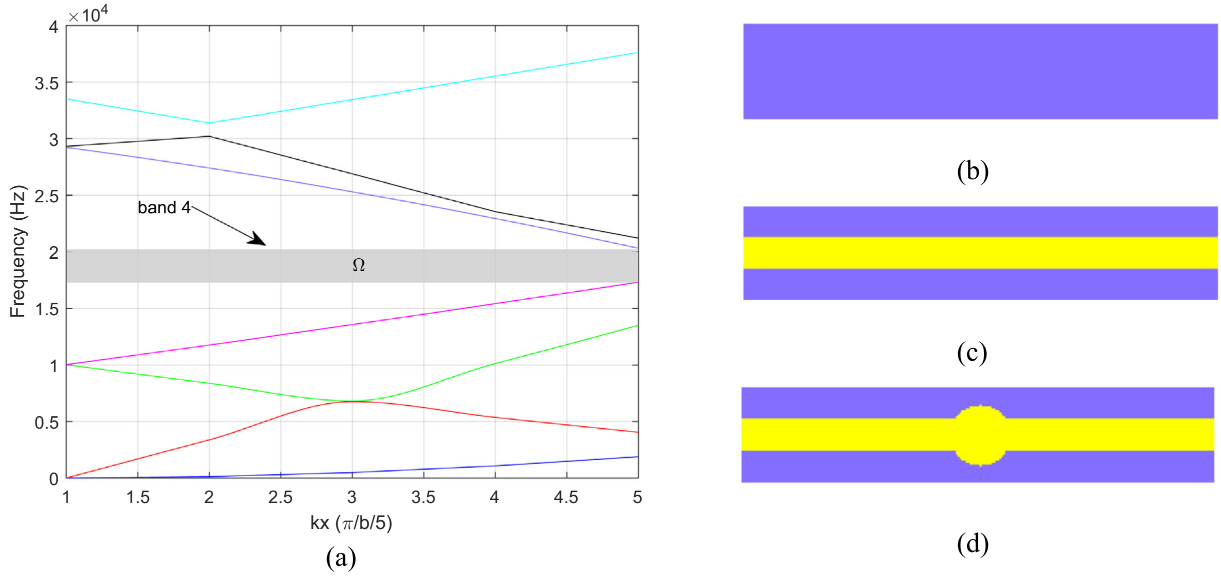


Fig. 4. The band structure and different unit cells.

Fig. 4(a) that this band gap is located between the 4th and 5th dispersion curves (count from bottom to top). For simplicity, this band gap is referred to as 'band 4', and the initial width is marked as Ω.

Fig. 5 shows the topological layout of unit cell in topology optimization for increasing the width of band 4. The corresponding band structure results are shown in Fig. 6. From Fig. 6, one can be found that the topological results of both unit cells increase the width of band 4 to varying degrees; the position and width of the optimized band 4 are shown in the shading areas, where the value shown in the shading areas is the width of the optimized band gap. From the perspective of optimal results, a wider band gap is obtained, the optimized width of band 4 increased 1.94 times and 2.34 times, respectively. The results prove the effectiveness of the proposed method for increasing the band gap width. Besides, another interesting outcome occurred, that is, the optimized band 4 is closer to the "flat band gap", which is more consistent with the band gap requirements in engineering applications.

5.2. Case II: Controlling the wave propagation at target speed

The purpose of this case study is to control the wave propagation at target speed, and the waves corresponding to the 6th and 7th dispersion curves are taken as examples to study the control method of the propagation of high-frequency waves. The optimized topological geometry is shown in Fig. 7, Among them, the numbers (a) and (b) result from the topology optimization problem with the 6th dispersion curve as the objective function, and the numbers (c) and (d) show the topological optimization problems related to the 7th dispersion curve. The slope equation of the corresponding dispersion curve is shown in Fig. 8.

It can be obtained from the slope curve of the dispersion curve in Fig. 8 that the proposed method can effectively control the slope of the dispersion curve, which means that the proposed method can realize the control of wave propagation. In addition, we can see that the slope curve in the Fig. 8 not only changes the amplitude of the slope, but also the trend of the slope, that is, the meaning expressed by the positive and negative values in Fig. 8, the positive value decreases, and the negative value increases. This point proves that the proposed method can achieve control of wave propagation at the desired target speed.

5.3. Case III: Limiting the propagation of low-frequency waves

This case explores the application of limiting the propagation of low-frequency waves, which is a problem that needs to be solved in engineering low-frequency vibration isolation. In this case study, two types of waves in the low-frequency range are selected, which are reflected in the dispersion curve, that is, the 2nd and 3rd dispersion curves. It should be emphasized here that this case study is an extension of case 2, and the target speed is zero, namely, what we want to achieve is that the waves cannot propagate in the structure.

Fig. 9 shows the topological layout of the unit cell in topology optimization for controlling wave propagation in low-frequency range, and their band structure results as shown in Fig. 10. Among them, the numbers (a) and (b) result from the topology optimization problem with the 2nd dispersion curve as the objective function, and the numbers (c) and (d) show the topological optimization problems related to the 3rd dispersion curve.

Comparing Fig. 4(a) and Fig. 10, it is easy to find that the slopes of the 2nd and 3rd dispersion curves have changed significantly (minimum to near zero), which means that the corresponding wave propagation speed in the periodic structure is almost zero, namely the proposed method can effectively limit the propagation of low-frequency waves. More meaningfully, we can find that in Fig. 10, with the slope of the dispersion curve decreases, new

Fig. 5. The optimized topological geometry.

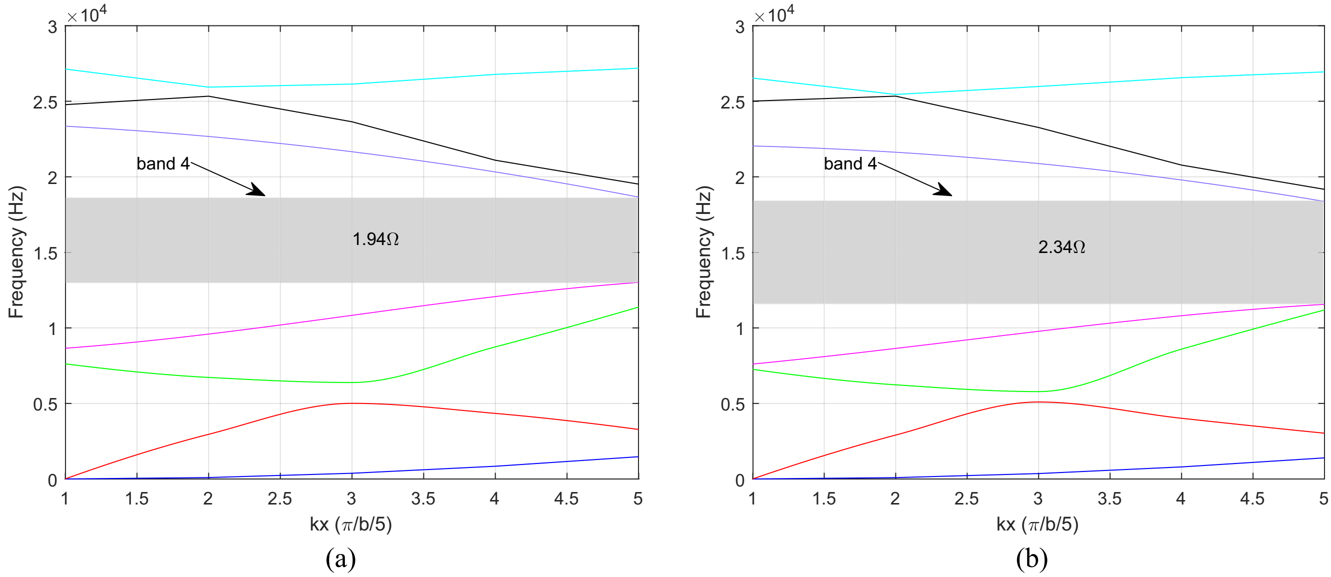


Fig. 6. The band structure related to the optimized topological geometry.

Fig. 7. The optimized topological geometry.

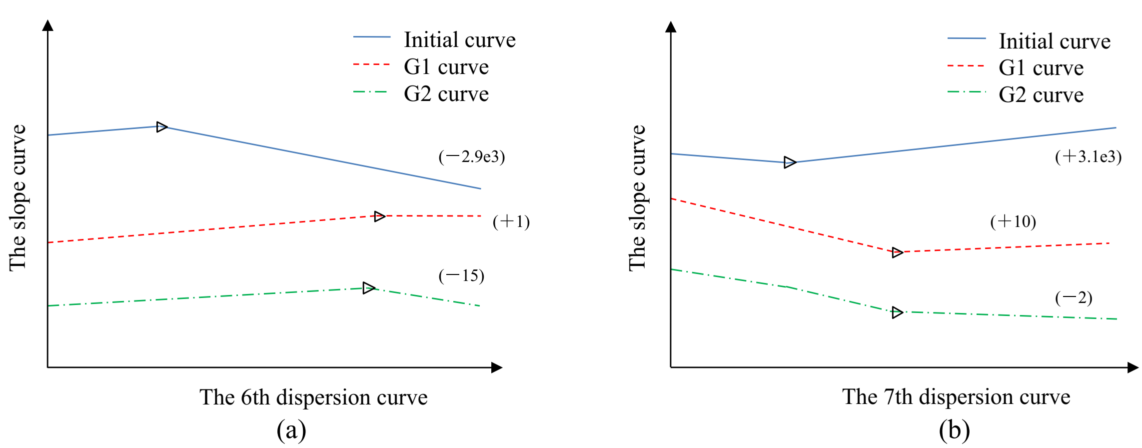


Fig. 8. The slope related to 6th and 7th dispersion curves.

low-frequency band gaps are created to varying degrees. This is evident in optimization problems where the 3rd dispersion curve is the objective function.

To further prove that the wave propagation speed can be controlled by changing the slope of the dispersion curve, a vibration frequency response of a periodic structure was carried out using experiment and finite element simulation. Comparing the variation of the dispersion curve in Fig. 10, the selected unit cell geometry is shown in Fig. 9(a). The reason is that this unit cell not affected by the newly generated band gap, which better verifies the wave propagation speed can be controlled. The experimental set-up is shown in Fig. 11. It should be noted that although the periodicity

Fig. 9. The optimized topological geometry.

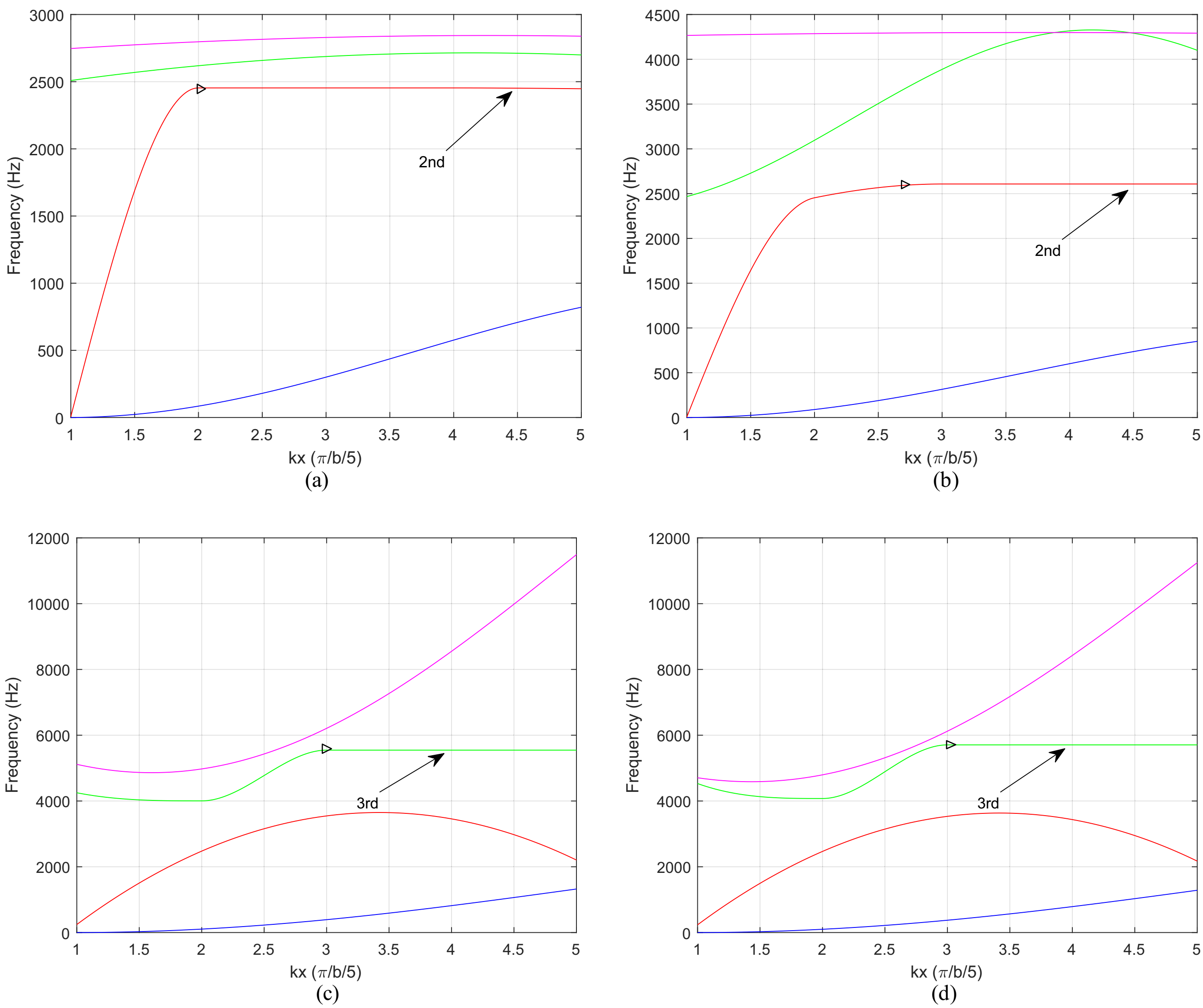
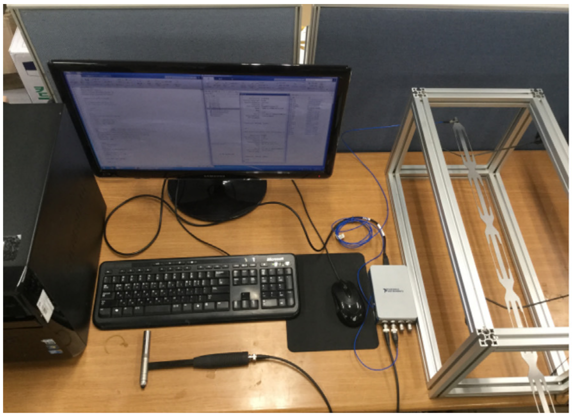


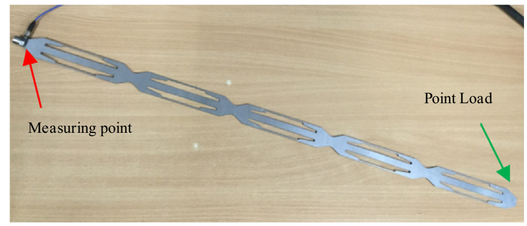
Fig. 10. The band structure related to the optimized topological geometry.

assumed in the numerical test is infinite, a limited number of unit cells are usually used to perform the approximate experiment supported by the theory. In this work, we manufactured 5 periodic unit structures to obtain the periodic structure, as shown in Fig. 11(b), which shows that simulation and the experiment already fit well. Of course, if it is possible, scholars could increase the periodic scale as much as possible to make the experiment result more accuracy.

For comparison, the same experiment was performed using finite element simulation (via Comsol Multiphysics), which is



(a)



(b)

Fig. 11. Experiment set-up: (a) experimental test, (b) periodic structure with 5 periods.

defined in the frequency domain of the solid mechanics interface. To do this simulation, two ends of the periodic structure are free, a total external force is applied at the left end, and the middle position in the *x*-direction of the right end is the measuring point of the frequency response. The frequency response function (FRF) is chosen as.

$$T = 20\log_{10}\left(\frac{A_c}{A_r}\right) \tag{11}$$

where A_c is the acceleration value at the measuring point, and A_r is a reference value, usually $A_r = 10e^{-5}m/s^2$.

By doing experiments and finite element simulation, the FRFs of the periodic structure generated by topology optimization are obtained, as shown in Fig. 12. It can be found in Fig. 12 that the FRFs obtained through experiments and finite element simulation is lower than 0 dB in the range of 850–2500 Hz, which means that the mechanical wave generated by vibration is limited in this range. Meanwhile, we can see that the range of FRFs results obtained by the above two methods (i.e., the simulation curve and experimental curve) is a better agreement with the range (850–2500 Hz) between the 1st and 3rd dispersion curves in Fig. 9(a).

However, from the perspective of comparison between experimental results and simulation results, one can find that there are some differences between the simulation curves and the experimental curves. The reason for this difference can be explained as some small changes in the periodic structure of the experiment. For manufacturing convenience, for example, the overall thickness of the periodic structure differs from the optimization result by 0.3 mm, the corner positions have been chamfered, and the processing error will cause the frequency range on the graph to change to a certain extent. However, the most important point that we cannot ignore is that both the above results prove that the propagation of the waves corresponding to the 2nd dispersion curve can be controlled by changing the slope of the 2nd dispersion curve, which further proves the effectiveness of the proposed method.

In addition, attention is placed on the dispersion curve obtained by the periodic structure of the wave propagating at the target speed. It is well known that the distance between two dispersion curve forms a band gap, and the existence of a band gap can control the propagation of waves, which is consistent with the research point in Case I. However, this band gap concept does not apply to Case II and Case III, which is special in this research. Take the 2nd dispersion curve in Fig. 9(a) as an example, although the dispersion curve exists in the range of 850–2500 Hz, it can be seen from the experimental and simulation results that the wave cannot propagate in this frequency range. The reason is that the wave propagation speed corresponding to the 2nd dispersion curve is controlled, which shown in the figure as the slope of the dispersion curve is zero.

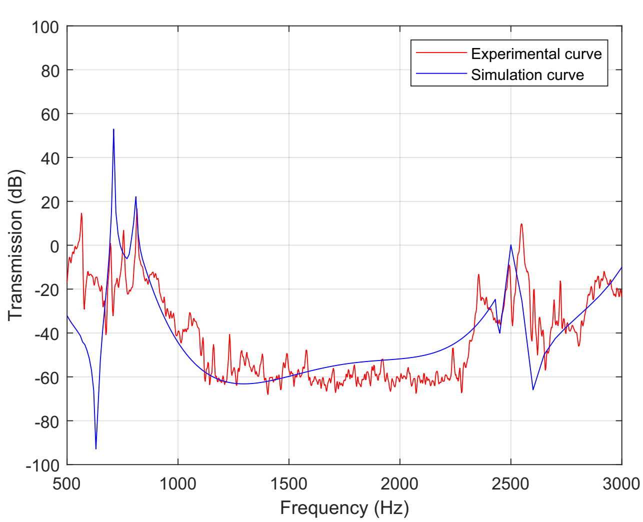


Fig. 12. FRFs of experiment and finite element simulation.

6. Conclusion and future work

This research proposed a multi-functional topology optimization for controlling wave propagation in a 1D structure, which can realize the control of wave propagation from two aspects: band gap and wave propagation speed. In this research, three case studies are considered: Case I: increasing the band gap width, Case II: controlling the wave propagation at target speed, and Case III: limiting the propagation of low-frequency waves. Accordingly, two objective functions are used to obtain topological results, one is the absolute band gap to be maximized, and the other is the slope of the dispersion curve to be controlled. The case study results prove the effectiveness of the proposed optimization method, even though the resulting clarity needs to be improved. More importantly, this method explores a new way to control the propagation of waves in periodic structures, for example, wave propagation at target speed and wave propagation are limited. Of particular concern is this method provides new insights into the low-frequency vibration isolation in engineering applications.

Future research to build upon this work will explore topology optimization for controlling the wave propagation in the desired direction, and multi-objective topology optimization that considers multi-application performance is also the focus of the next step to develop methods more suitable for practical engineering applications. In addition, the problem of eigen-frequencies are equal that may occur when solving the adjacent eigenmode are also worthy of further study.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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