Composite Structures 331 (2024) 117859

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Mechanical vibration absorber for flexural wave attenuation in multi-materials metastructure

Long Liu [a,b], Ji Wan Kim [b], Gil Ho Yoon [b,*], Bing Yi [a,*]

[a] School of Traffic and Transportation Engineering, Central South University, Changsha, 410075, China
[b] School of Mechanical Engineering, Hanyang University, Seoul, 04763, South Korea

ARTICLE INFO

Keywords:
Phononic crystal
Flexural wave attenuation
Composite sandwich
Mechanical vibration
Band gap
Metastructure

ABSTRACT

Vibration isolation is a promise to suppress mechanical vibration from a host structure, similarly, a mechanical vibration absorber, a simple but effective device to attenuate flexural wave propagation, which has been implemented in civil and mechanical engineering. This paper presents a type of composite sandwich phononic crystal to attenuate the flexural wave propagation in a beam structure, which can effectively suppress mechanical vibration in a broad band gap by repetitively arranging phononic crystal. First, the elastic wave dispersion characteristic in a composite sandwich beam structure is derived, and a triangular shape phononic crystal for flexural wave attenuation by taking advantage of destructive interference is presented. Then two dimensional phononic crystals are designed by assembling four different unit-cells of metabeam. Finally, numerical experiments are conducted to verify the effectiveness of the proposed mechanical metamaterial absorbers to attenuate flexural wave propagation, the numerical results indicate that the proposed metamaterial is of good performance in mechanical vibration suppression, which can effectively mitigate structure vibration in low-frequency domain than the structure without phononic crystal and single layer metamaterial beam structure. It is the first attempt to design a mechanical metamaterial absorber with the mechanism of destructive interference with composite sandwich phononic crystal.

1. Introduction

This research presents flexural wave attenuation in multi-material metastructure. Mechanical vibration is a prevalent occurrence in nature, extensively observed in domains such as aerospace, railway infrastructure, civil engineering, and precision instruments [1–5]. However, vibrations often lead to product fatigue and reduce the lifespan of mechanical components. In more severe cases, they can pose substantial risks to life and property safety. Therefore, thoroughly studying mechanical vibration absorbers to suppress the propagation of flexural waves in industrial settings holds paramount importance. To contribute this topic, the multi-material metastructure relying on the destructive interference phenomenon is presented.

Many relevant researches about metastructure exist. Utilizing the principle of Bragg scattering and local resonance, the phononic crystal and metamaterial, a kind of structure with periodically arranged unit cells [6–8], have received much attention because of their prospective advantages in acoustic cloaking [9–11], energy harvesting devices [12,13], and wave manipulation [14]. Inspired by the Maltese-Cross shape, Panahi et al. [15] numerically and experimentally studied the wave dispersion characteristics of two types of unit cells, and further constructed a metaplate to suppress the wave propagation in plate. Li et al. presented a pressure-resistant cylindrical shell cover comprising graded zero Poisson's ratio metamaterials for vibroacoustic attenuation in [16]. In [17–19], topology optimization is introduced to improve phononic crystal band gap design, which can effectively attenuate wave propagation by reasonably distributing two different materials. Liu et al. [20] presented a gradient-based method to manipulate the wave propagation in one-dimensional structure, which not only can increase the bandgap width but also the wave propagation speed can be further controlled. Li et al. [21] proposed a new topology optimization pipeline to investigate the materials distribution of 3D phononic crystals based on the fixed-grid finite element method (FEM) and evolutionary procedure. The aforementioned optimization methods can effectively design phononic crystals to attenuate wave propagation with reasonable materials configuration, however, the density-based approach inevitably introduces grey elements which sometimes cause significant alternations and deviations on the performance and response of the body-fitted designs postprocessed from the optimized layouts with grey elements. Overcoming this side effect is pretty challenging and difficult.

* Corresponding authors.
E-mail addresses: ghy@hanyang.ac.kr (G.H. Yoon), bingyi@csu.edu.cn (B. Yi).

https://doi.org/10.1016/j.compstruct.2023.117859
Received 11 August 2023; Received in revised form 24 October 2023; Accepted 27 December 2023
Available online 29 December 2023

Many relevant researches with local resonators exist. Local resonator, taking advantage of resonant behavior in each unit chamber, shows promising performance in manipulating elastic waves with low-frequency excitation. Shui et al. introduced an equivalent stiffness technique to design mechanical vibration absorbers with tunable piecewise-linear stiffness, which can realize a large optimum vibration-absorption range and a wide suppression band in [22]. Pai et al. presented a metamaterial beam consisting of a uniform isotropic beam and mass–spring-damping vibration absorbers for broadband absorption of transverse elastics wave, which reveals the working mechanism of the absorber in [23]. In [24], Cai et al. proposed a metamaterial beam with embedded quasi-zero-stiffness resonators for suppressing flexural wave propagation in low-frequency band gaps, both the numerical analysis and experiment demonstrate the effectiveness of the present metamaterial beam in the application for wave attenuation. In [25], Xu et al. designed a honeycomb hierarchical lattice with embedded rubber-coated resonators for wave attenuation at subwavelength scales in two separate frequency regions, the experimental result shows a good agreement with the numerical analysis. Based on Hamilton's principle and the energy averaging method, Jin et al. constructed an analytical model to design a multifunctional metastructure for suppressing mechanical vibration by combining a honeycomb sandwich structure and a locally resonant metastructure in [26]. Wang et al. theoretically and numerically investigated the flexural wave attenuation in an Euler–Bernoulli beam with lateral local resonators, which reveals that the lateral local resonators can smooth and lower the response peaks at the sacrifice of the band gap effect in [27]. Miranda et al. analytically, numerically, and experimentally studied flexural wave propagation in an elastic metamaterial thick plate with periodic arrays of spring–mass resonators and further investigated the effect on the formation of Bragg-type and locally resonant band gaps in [28]. Casalotti et al. studied an Euler–Bernoulli beam with a distributed array of nonlinear spring–mass vibration absorbers in [29]. Banerjee et al. proposed a graded metamaterial beam for attenuating flexural wave propagation, which achieved a wider band gap without increasing the mass of resonators in [30]. In [31], Zhang et al. proposed a rose-shaped mechanical metamaterial by shape morphing method. Dudek et al. presented innovative microscopic 2D and 3D functionally-graded mechanical metamaterials capable of exhibiting a broad range of Poisson's ratio depending on composition in [32]. The local resonance metamaterial is of extraordinary capability in low-frequency elastic wave manipulation, nevertheless, widening the broad band gap for elastic wave attenuation in a beam structure still remains a challenge, which limits the flexibility and robustness in industry applications.

In recent years, a few researchers attempted to design metamaterial beams by taking advantage of destructive interference (DI), which is also a promising and effective method to analytically design metamaterial and phononic crystal with a broad band gap for controlling elastic wave propagation. In [33], Xu et al. proposed a coupled phononic crystal plate with a broadband gap to suppress elastic wave transmission by taking advantage of destructive interference, the proposed system can online adjust the frequency range of wave attenuation by modifying the shunt circuits. In [34], Fu et al. designed a new metamaterial plate with acoustic black hole structures and destructive interference structures, which can not only suppress mechanical vibration but also can be used for noise reduction. In [35], based on Euler–Bernoulli beam theory, Yoon et al. presented triangular shape mechanical metamaterial absorbers to attenuate bending wave propagation, both the numerical analysis and experimental result demonstrate the performance on wave attenuation. However, based on destructive interference theory in [36–39], it is known that the control of the mechanical vibration within a low-frequency range is challenging due to the limitation of the size of the phononic crystal. The inadequate investigation of mechanical metamaterial absorbers with destructive interference urges this work which is the first attempt to design a metamaterial beam with two-dimensional multilayer phononic crystals.

To effectively design mechanical metamaterial absorbers with a broad band gap to attenuate the flexural wave propagation in a beam structure, this study presents a *composite sandwich-based functionally gradient phononic crystal* with the mechanism of destructive interference for absorbing mechanical vibration. The wave dispersion characteristic of a multilayer Euler–Bernoulli beam and the derived flexural wave propagation theory in the composite sandwich beam are first investigated. Subsequently, 2D triangular shape units are designed and functionally graded phononic crystals are assembled for suppressing mechanical vibration in a broad band gap. Finally, numerical experiments are conducted to demonstrate the effectiveness of the presented composite sandwich-based vibration absorber to attenuate flexural wave propagation in the low-frequency range. The main contributions of this paper can be summarized as follows:

1. A novel and improved mechanical vibration absorber for attenuating flexural wave propagation in the metamaterial beam is introduced. To the best of our knowledge, it is the first attempt to control flexural wave propagation in a beam using composite sandwich-based phononic crystal.
2. Compared with single phase-based phononic crystal for suppressing mechanical vibration, the size of the proposed composite sandwich-based phononic crystal can be smaller when the frequencies of elastic waves of interest are same.
3. The presented composite sandwich-based phononic crystal demonstrates comparable performance in absorbing flexural waves to that of a single material-based absorber, both at lower and higher frequencies.

The rest of this paper is organized as follows: Section 2 introduces the theory of flexural wave propagation in metamaterial beam and the basic rules of phononic crystal design. Then the numerical experiments are conducted to demonstrate the effectiveness and advancement of the proposed composite sandwich phononic crystal structure in Section 3. Finally, the conclusion and future work are presented in Section 4.

2. Phononic crystal with destructive interference

In this section, we first take the theory of the Euler–Bernoulli beam as an example to establish the mathematical model for describing the flexural wave propagation in composite sandwich beam, and then the triangular shape phononic crystal is presented to attenuate the flexural wave propagation with the mechanism of destructive interference.

2.1. Flexural wave in multi-materials beam

The governing equation of the flexural wave propagation in a metamaterial beam can be formulated as follows:

$$\frac{\partial^4 \Psi}{\partial x^4} + \frac{M}{D}\frac{\partial^2 \Psi}{\partial t^2} = 0 \tag{1}$$

where the bending displacement of the beam is denoted by Ψ. The mass of the section is denoted by $M = \sum_i \rho_i A_i$. The mass density and the cross-sectional area and the bending stiffness are denoted by ρ, A, and $D = \sum_i E_i I_i$, respectively. Young's modulus and the moment of inertia are denoted by E and I, respectively. The speed of the flexural wave propagating in a metamaterial beam can be formulated as Eq. (2), where ω is the angular frequency, and the subscript i denotes the total members in a section where the number of the cross sections is n.

$$c = \left(\frac{\sum_i E_i I_i}{\sum_i \rho_i A_i}\right)^{\frac{1}{4}} \omega^{\frac{1}{2}} \quad (i = 1, 2, 3\cdots, n) \tag{2}$$

By using the relationship between the wave speed c and frequency f, the flexural wavelength of a beam structure with composite material is formulated as Eq. (3). Fig. 1 shows the schema of a multilayer structure of a section, which is constructed by three members.

$$\lambda = 2\pi \left(\frac{\sum_i E_i I_i}{\sum_i \rho_i A_i}\right)^{\frac{1}{4}} \omega^{-\frac{1}{2}} \quad (i = 1, 2, 3\cdots, n) \tag{3}$$

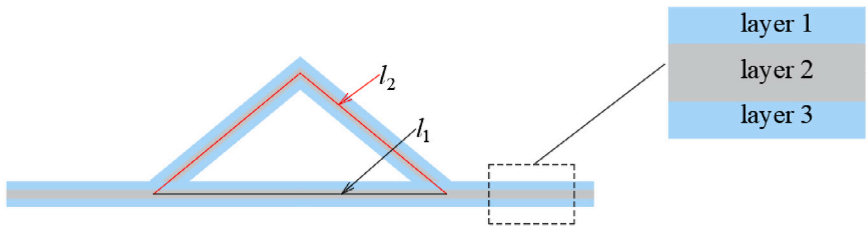
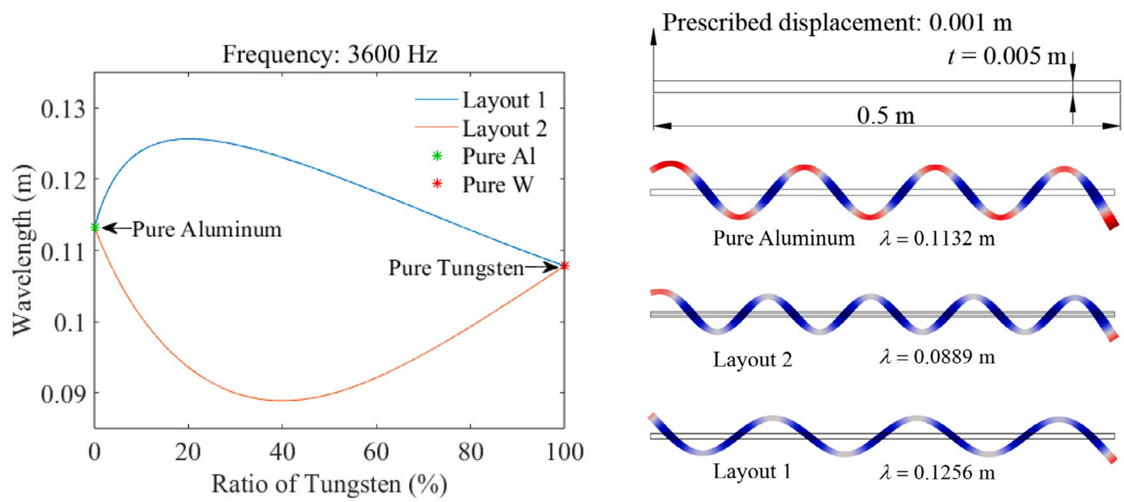


Fig. 1. An example of destructive interference of a multilayer-based phononic crystal.



(a) (b)

Fig. 2. An example of flexural wave propagation in a beam structure. (a) The relationship between the material distribution and wavelength and (b) the geometric shape of a beam structure and its boundary condition (E^{Al}=70 GPa, ρ^{Al}=2700 kg/m3, ν^{Al}=0.33, E^{W}=411 GPa, ρ^{W}=14,300 kg/m3, ν^{W}=0.28, f^{target}=3600 Hz) and the structure responses of a beam with different material layouts, where Al and W indicate Aluminum and Tungsten, respectively.

It can be found that the bending wave propagation is similar to acoustic and electromagnetic to some degree. The wavelength of the flexural wave is related to the physical properties of the medium such as Young's modulus and density. The difference lies in the wavelength of the bending wave depending on the source frequency of excitation that is called the wave dispersion phenomenon. In other words, the wavelength of flexural wave propagating in a beam can be changed as the material property changes. As we know that the size of the phononic crystal can be reduced if the wavelength of the flexural wave can be further decreased [35], which inspires us to further study the physical characteristic of the flexural wave propagation in a metamaterial beam in this research. The details for deriving the governing equation of the flexural wave propagation in multi-materials beam can be found in the appendix. For wave separation mechanism in geometry conjunction, interested readers can refer to the Refs. [40–42].

From the theory, an imminent idea to decrease the wavelength may be reducing the bending stiffness of the cross-section and increasing the mass property of the section, however, as for a given section, the mass M and bending stiffness D become constants when using single material or layer, which intrigue this paper studying the physical characteristic of flexural wave propagation in a composite sandwich beam. Therefore, this paper limits the usage of materials to two kinds of material, i.e., aluminum and tungsten, in order to illustrate the advancement of the multilayer metabeam. To quantitatively measure the wavelength of the composite sandwich beam with various material configurations, the response of a composite beam structure with a total thickness of 0.5 mm under the excitation of 3600 Hz source frequency is recorded in subplot (a) in Fig. 2, which exploits the relationship between the thickness of aluminum and tungsten, where layout 1 means both the layer 1 and 3 are made of tungsten and layer 2 is made of aluminum. The opposite material distribution is layout 2. The green and red * show the wavelengths of the structures with pure aluminum and tungsten, respectively. To intuitively display the wavelength difference of various material layouts in a composite beam structure, the subplot (b) in Fig. 2 shows an example of the bending wave. It can be clearly found that the wavelength of layout 1 is the longest among the structure responses, and the wavelength of the layout 2-based beam is shorter than one material-based beam structure. Numerical analysis indicates that the presented composite sandwich beam structures show promising performance in manipulating the wavelength of a flexural wave under transverse vibration. By arranging the layout of the two materials, the wavelength can be reduced by approximately 20% compared with conventional single material beam structure, and it can also be increased by 10% on average.

2.2. Phononic crystal designing

Destructive interference is a common phenomenon in nature, which refers to a type of interference in which two interfering waves have

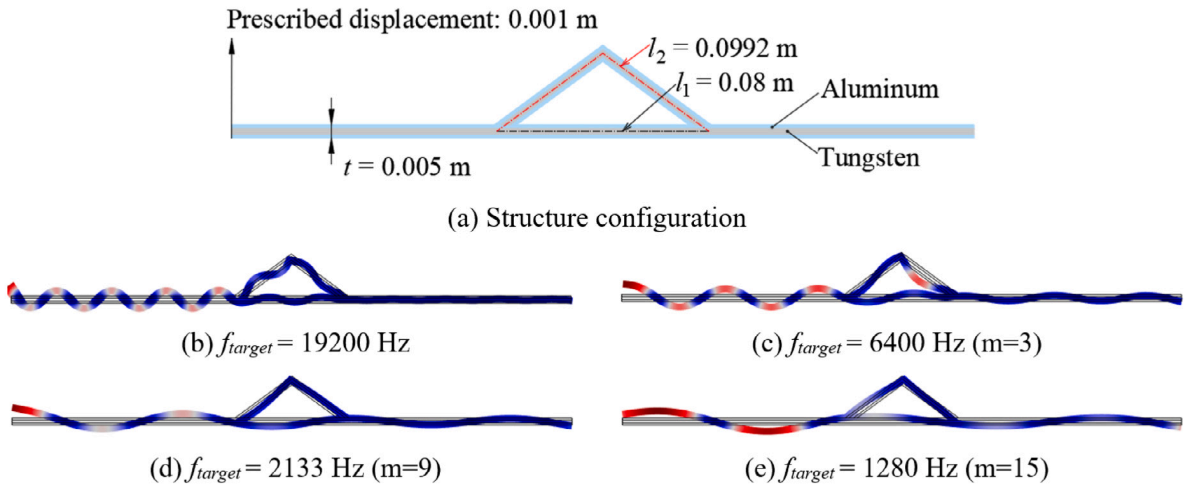


Fig. 3. A beam structure with phononic crystal.

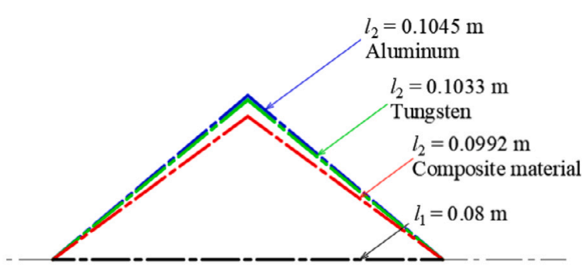


Fig. 4. Neutral axes of phononic crystal for attenuating flexural wave propagation under excitation with 19,200 Hz.

a displacement in opposite directions. It occurs when waves come together so that they can cancel each other out. In other words, the amplitude of the resultant wave will be decreased due to destructive interference. Inspired by the mechanism of destructive interference, the advantage can be further applied to attenuate the flexural wave propagation with the present composited structure in Fig. 1.

To design an effective mechanical vibration absorber with destructive interference, a triangular shape-based phononic crystal is considered to attenuate the flexural wave propagation. A phononic crystal with destructive interference is presented in Fig. 1. The black line is the short path of the triangular shape that is denoted by l_1, while the long path depicted by a red line is denoted by l_2. The flexural wave will separate into two branches of a wave at the bifurcating node and propagate forward along the paths. Due to the length difference between the long and short paths, the phenomenon of destructive interference will occur and suppress the propagation of the bending or transverse wave at the conjunction node. To realize the destructive interference for manipulating the bending wave propagation in a composite sandwich beam, the short path l_1 and long path l_2 can be determined by the following rules:

$$\delta = l_2 - l_1 = (n + \frac{1}{2})\lambda, (n = 0, 1, 2, \cdots) \tag{4}$$

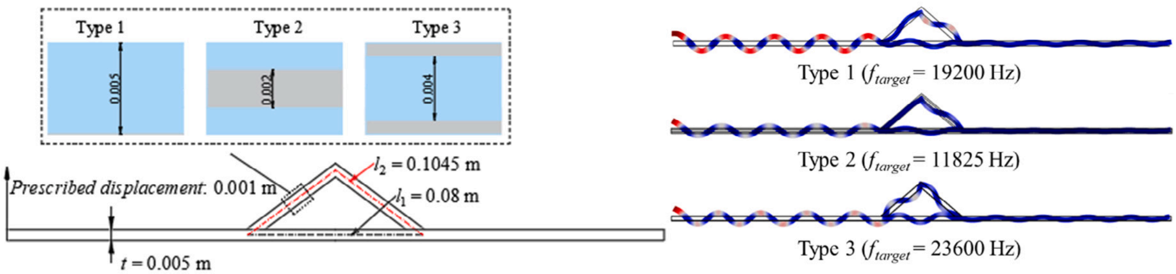
where δ denotes the length difference. It can be observed that the length difference between the long and short paths is related to wavelength.

Fig. 3 shows an example of a beam structure with a phononic crystal for attenuating the flexural wave propagation under transverse vibration at 19,200 Hz. The value, 19,200 Hz, is not a threshold value. The finite element model without a damping is employed to show the validation (Eqs. (4) and (5)) for the destructive interference. However, in a real engineering structure with damping, the vibration at that high frequency is suppressed by damping. The subplot (a) in Fig. 3 gives the geometric shape of the beam structure and its boundary condition, where the short and long paths of the phononic crystal are set to 0.08 m and 0.0992 m, respectively. The total thickness of the beam structure is 0.005 m, in which the layer 1 and layer 3 are aluminum sheets whose thicknesses are 0.0015 m, and the layer 2 is Tungsten. The subplot (b) in Fig. 3 showing the structural response of the phononic structure shows that the presented composite sandwich phononic crystal can effectively suppress the flexural wave at 19,200 Hz. In addition, the present crystal structure can attenuate the other flexural waves satisfying the Eq. (5). The subplots (c)–(e) in Fig. 3 show the structure responses under transverse vibration with source frequencies of 6400 Hz, 2133 Hz, and 1280 Hz, respectively. Note that the present phononic crystal can be used to attenuate flexural waves with higher frequencies than 19,200 Hz by Eq. (4).

$$\lambda^* = m\lambda, (\mathrm{m} = 1, 2, 3, \cdots) \tag{5}$$

As illustrated in Fig. 2, it is possible to use different material distributions to improve the performance. For an example, Fig. 4 shows the neutral axes of phononic crystal with different material layouts for attenuating the flexural wave propagation. It can be found that the size of the phononic crystal can be further reduced using a sandwich composite beam rather than a single material. To show this characteristic further, another example is considered in Fig. 5. Fig. 5(a) shows one single layer (Type 1) and two composites (Types 2, 3) and Fig. 5(b) illustrate the structural response of the metamaterial beams with the same size of the phononic crystal. Compared with the single layer-based metastructure, the present composite metabeams can effectively attenuate mechanical vibrations at various frequencies by adjusting the thickness values of the layers and materials.

In addition, another example is considered to illustrate the benefit of the present composite DI structure in Fig. 5. Fig. 5(a) shows the three types of material layout. The layout 1 is a typical single layer-based metabeam while the types 2 and 3 are multilayer-based metamaterial beam structures. Fig. 5(b) shows the responses of the three phononic crystals with the same size. The frequencies in Fig. 5(b) are computed



(a) Material layout (b) Structure response

Fig. 5. Phononic crystal with different material distribution for attenuating mechanical vibration.

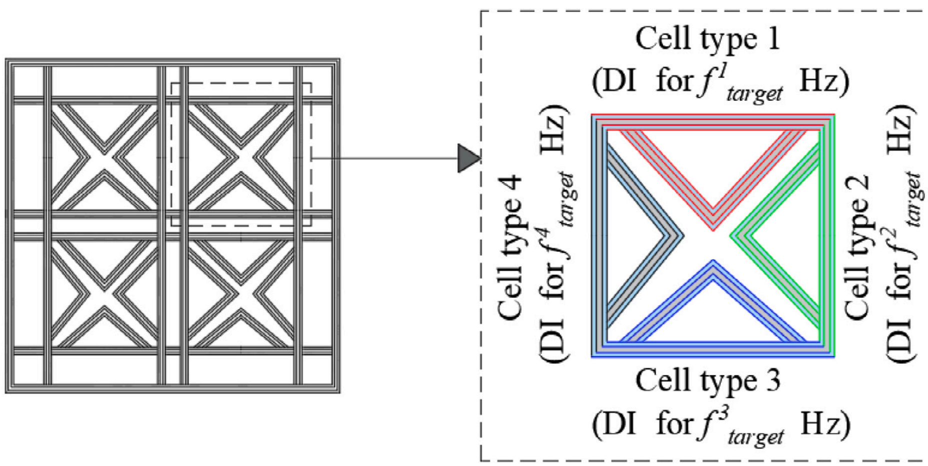


Fig. 6. Schema of a rectangular phononic crystal.

by setting 1 for m. In our simulations, the significant decreases in the responses can be observed. However, the reduction of the magnitude cannot be estimated prior to the finite element simulation. By investigating these results, it turns out that the present composite structure, i.e., Type 2, is effective at a lower frequency, which is attractive from the point of view of industrial application.

From the engineering point of view, the working frequency of industrial products lies in a range rather than a specific frequency, thus, the structure should be of the capacity to attenuate the elastic wave propagation in a broadband gap. Therefore, a rectangular framework with four triangular phononic crystals which can suppress mechanical vibration is proposed in Fig. 6. By alternating the distances, the four types of Type 2 are presented.

3. Experiments and validation

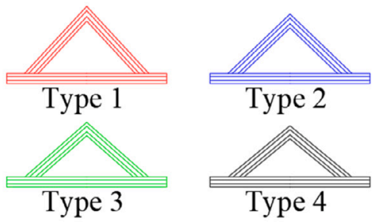
This subsection shows the response of the structure with/without phononic crystals under harmonic force excitation, and acceleration of the structure is used as the indicator to verify the effect of the phononic crystal for suppressing the mechanical vibration of the multilayer metamaterial beam structure. If not otherwise specified, the material distribution in the cross section of all the examples are same with the type 2 cross section shown in Section 2, and all the numerical simulations are carried out in COMSOL Multiphysics.

3.1. Example 1

To illustrate the effectiveness of the phononic crystal for manipulating the bending wave propagation, the four typical phononic structures are constructed, of which the structure patterns are presented in Fig. 7, and the specific parameters for phononic crystal are given. Fig. 8 shows the boundary conditions of the structures, subplots (a) and (b) show the structures without phononic crystals which are introduced for comparison, the material distribution in cross section of structure 2 is aluminum. A unit force in both x and y axes is applied and the acceleration at the right side of the structure is measured.

The structure responses under various excitations are shown in Fig. 9, it can be observed that the structures without phononic crystal vibrate more violent than the designed phononic crystal structure, especially in the designed bandgaps, which shows that the designed phononic crystal is of good performance on suppressing the mechanical vibration of the structure. In addition, the designed phononic crystal can create the bandgap to attenuate the flexural wave propagation and it can be clearly found that the acceleration of the right of the structure in the designed bandgap is smaller than other source frequencies.

To further validate the advancement of the designed phononic crystal structure, we measure the acceleration of both the upper and bottom lines of the structure under point load excitation with a certain source frequency, 1000 Hz, the structure configurations are given in



(a)

	l_1 (m)	l_2 (m)	δ (m)	f_{target} (Hz)
Cell type 1	0.06	0.0881	0.0281	600
Cell type 2	0.06	0.0829	0.0229	900
Cell type 3	0.06	0.0799	0.0199	1200
Cell type 4	0.06	0.0778	0.0178	1500

(b)

Fig. 7. Geometry of four different cell types of the composite sandwich phononic crystal (b) the details of the geometric parameters. (n = 0 and m = 15).

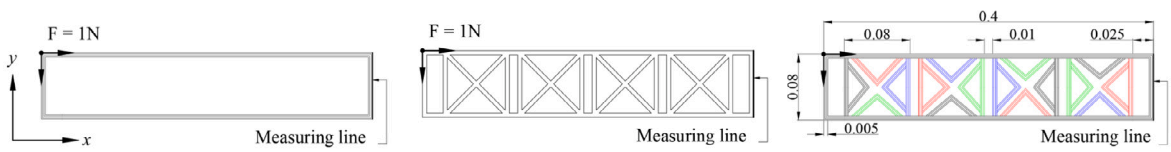


Fig. 8. Boundary conditions of the structures, (a) Structure 1 without PnCs, (b) Structure 2 without PnCs, (c) Structure with PnCs.

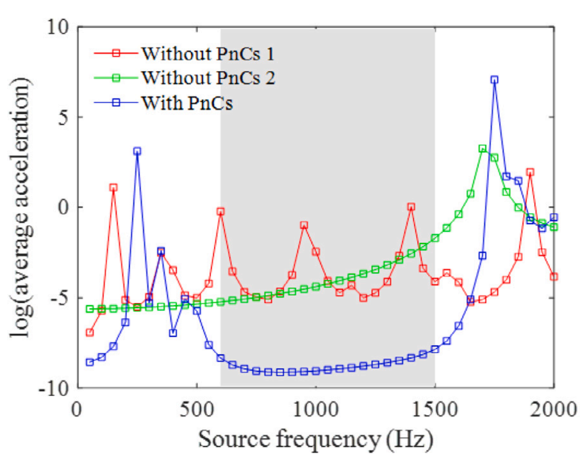


Fig. 9. Structure response under various source frequencies.

Fig. 10. Fig. 10(b) shows the structure response under the excitation with 1000 Hz transverse force. Intuitively, the structure with phononic crystal can effectively control the flexural wave propagation when compared with the structure without phononic crystal, with the help of the phononic crystal, the bending wave is effectively attenuated at the first unit cell while the bending wave directly propagates in the structure without phononic crystal. To quantitatively measure the effect of the phononic crystal on suppressing the structure vibration, 40 points on both the upper and bottom lines of the structure are used to measure the acceleration of the structure, and Fig. 10(c) shows the acceleration of the upper and bottom lines of the structures, it can be found that the structure with phononic crystal can suppress the flexural wave propagation as the acceleration of the structure is of a downward trend when the measurement point far away from the load point.

As we emphasize that the present composite sandwich phononic crystal can attenuate wave propagation under the excitation with both the lower and higher source frequencies than single-phase phononic crystal, we adopt the structure shown in Fig. 8(b) as the object and apply different material configurations to demonstrate the advancement of the proposed composite sandwich phononic crystal. Fig. 11 shows the structure response under the excitation of the source frequency lies in the range between 0 Hz and 3000 Hz, we can find that bandgap of the single material-based phononic crystal lies in the range of 750 Hz and 2450 Hz, and the bandgap of type 1 based phononic crystal lies in the range of 500 Hz and 1700 Hz, and the bandgap of type 2 based phononic crystal lies in the range of 800 Hz and 2700 Hz. It can be concluded that the proposed composite sandwich-based phononic crystal can realize attenuation of the mechanical vibration under the excitation with a lower source frequency, which demonstrates the advancement of the presented composite metabeam method.

3.2. Example 2

Another L-shape structure is introduced to illustrate the effectiveness of the composite sandwich phononic crystal for attenuating the flexural wave propagation in a beam-like structure, which is shown in the first row of Fig. 12. Similar to example 1, we first verify the performance under excitation with a range of source frequency, the second row of Fig. 12 shows the average acceleration of the structure of the bottom line, we can find that the response of the structure with phononic crystal is smaller when the source frequency lies in the range of bandgap, which indicates that the proposed composite sandwich phononic crystal can effectively manipulate the wave propagation at the designed frequency range.

Fig. 13 shows the structure response under source excitation with a frequency of 1000 Hz. Fig. 13(a) presents the measuring lines for the evaluation, and Fig. 13(b) presents the structure deformations under the excitation. It can be observed that the structure without phononic crystal vibrates heavily and the phononic crystals can effectively attenuate the mechanical vibration. Fig. 13(c) and (d) quantitatively

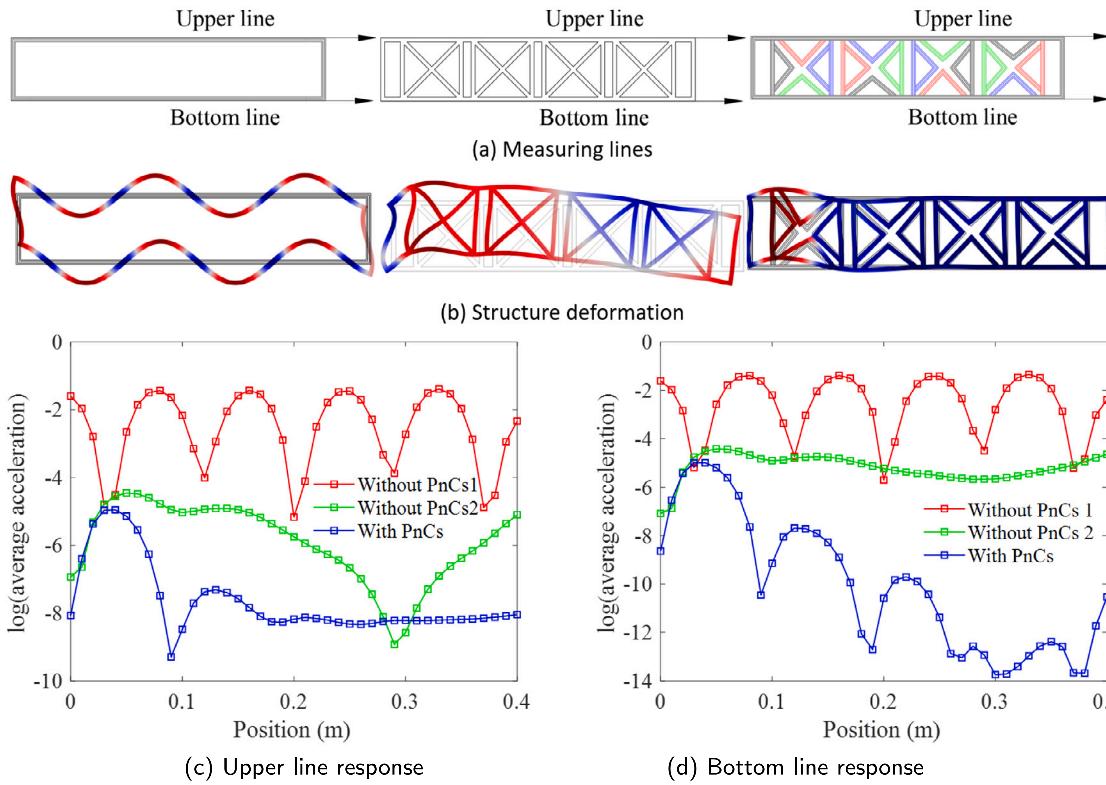


Fig. 10. Structure deformation and acceleration at 1000 Hz.

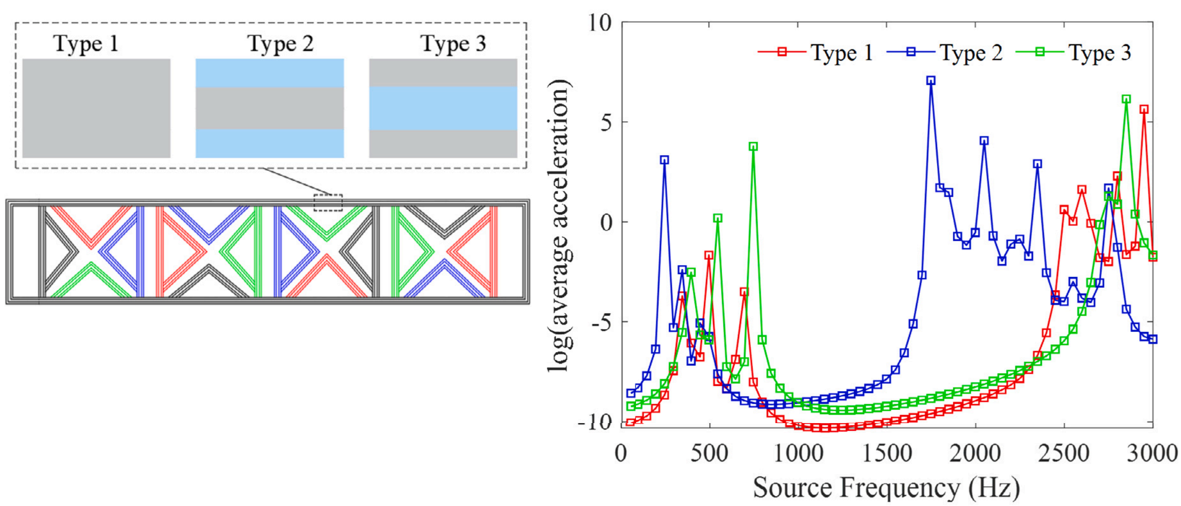


Fig. 11. Structure response with different material configuration(Type 1: Pure Tungsten, Type 2:Layer 1 and Layer 3 for Aluminum with 0.0015 m, respectively; Type 3: Layer 1 and Layer 3 for Tungsten with 0.0015 m, respectively).

show the structure responses along the upper and right lines, which indicate that flexural wave is greatly attenuated in both the upper and right beams of the structure when using phononic crystal. Similarly, Fig. 14 indicates that the present phononic crystals are of good ability to suppress mechanical vibration under the excitation of a low source frequency. Overall, we can make a conclusion that the phononic crystal can effectively absorb mechanical vibration.

3.3. Example 3

We extend the study of example 1 to a rectangular structure by repetitively arranging phononic crystals, which is shown in the first row of Fig. 15, similarly, the presented phononic crystals are of capacity to suppress mechanical vibration in a broadband gap which can be found in the second row of Fig. 15. The first row of Fig. 16

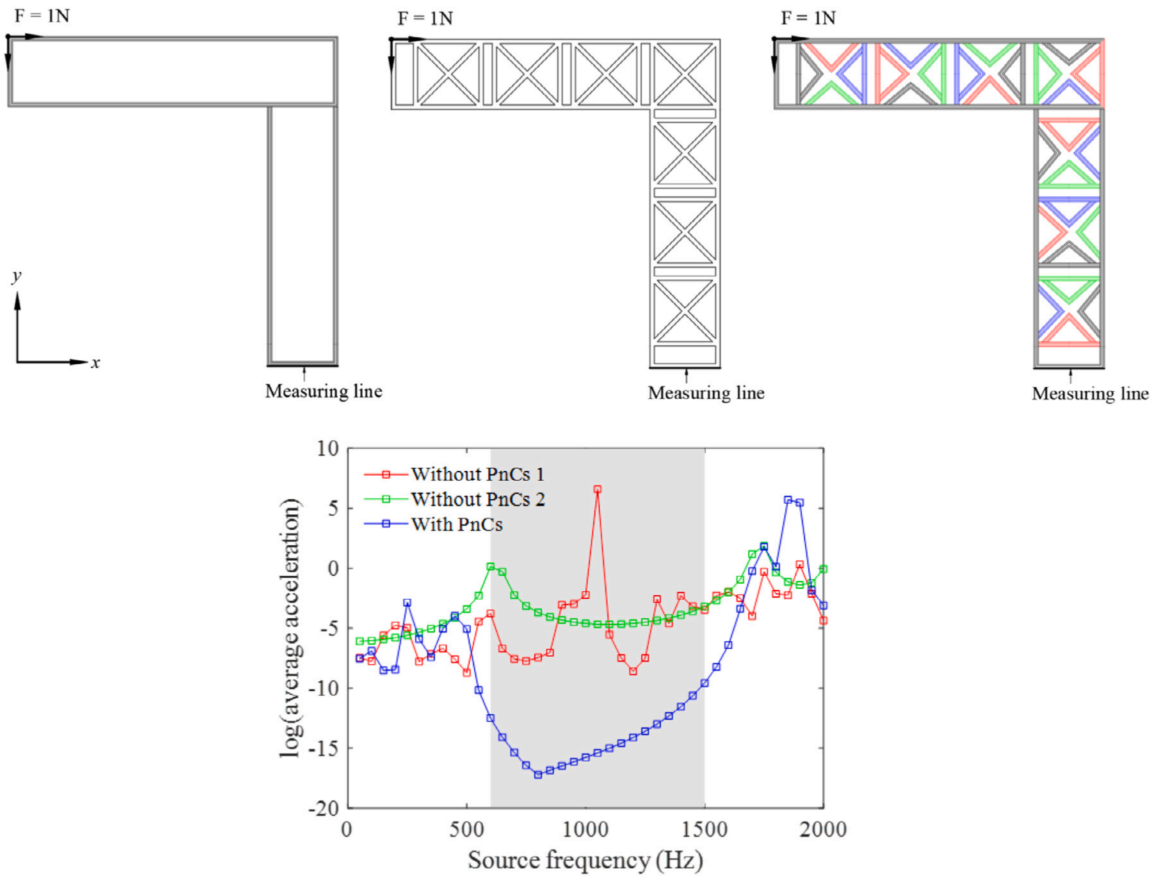


Fig. 12. Structure response under various source frequencies.

shows the structure response under 1000 Hz source frequency, and we can find that the vibration of both the bottom and right lines of the structure is controlled. In addition, Fig. 17 shows the bandgap of the structure with different material settings, we can find that the proposed metamaterial can attenuate the flexural wave propagation in a lower source frequency. Overall, it can be concluded that the introduction of phononic crystals can manipulate the flexural wave propagation in a beam structure, it is of good performance in attenuating mechanical vibration.

4. Conclusion and future work

This paper presents a novel multi-layer metamaterial beam for mechanical vibration suppression by taking advantage of destructive interference, which can effectively attenuate flexural wave propagation in a broad band gap through repetitively arranging gradient index phononic crystal. First, the physical characteristic of the multi-layer metamaterial beam is derived and demonstrated, and then the triangular shape phononic crystal with a mechanism of destructive interference is presented to attenuate the flexural wave propagation. Additionally, a two-dimensional gradient index phononic crystal set is designed for elastic wave manipulation in a broad band gap. Finally, numerical examples are further conducted to illustrate the effectiveness of the proposed multi-layer metamaterial for suppressing mechanical vibration in a beam structure.

The proposed multi-layer phononic crystal for flexural wave attenuation can be widely used in the design of the key components for mechanical products and building construction, which can greatly suppress the mechanical vibration in a broad range by reasonably designing the phononic crystal. In further work, the advancement of the multi-layer metamaterial can be applied to the plate structure and multiphysics coupling problems.

CRediT authorship contribution statement

Long Liu: Methodology, Writing – original draft. **Ji Wan Kim:** Data curation, Validation. **Gil Ho Yoon:** Funding acquisition, Conceptualization, Supervision. **Bing Yi:** Funding acquisition, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgments

This work was funded by the Korea Institute of Energy Technology Evaluation and Planning, South Korea under Grant No. 20202020800030, National Research Foundation of Korea, South Korea under Grant No. NRF-2019R1A2C2084974, the National Natural Science Foundation of China under Grant No. 51975589. This work was also supported by China Scholarship Council (CSC).

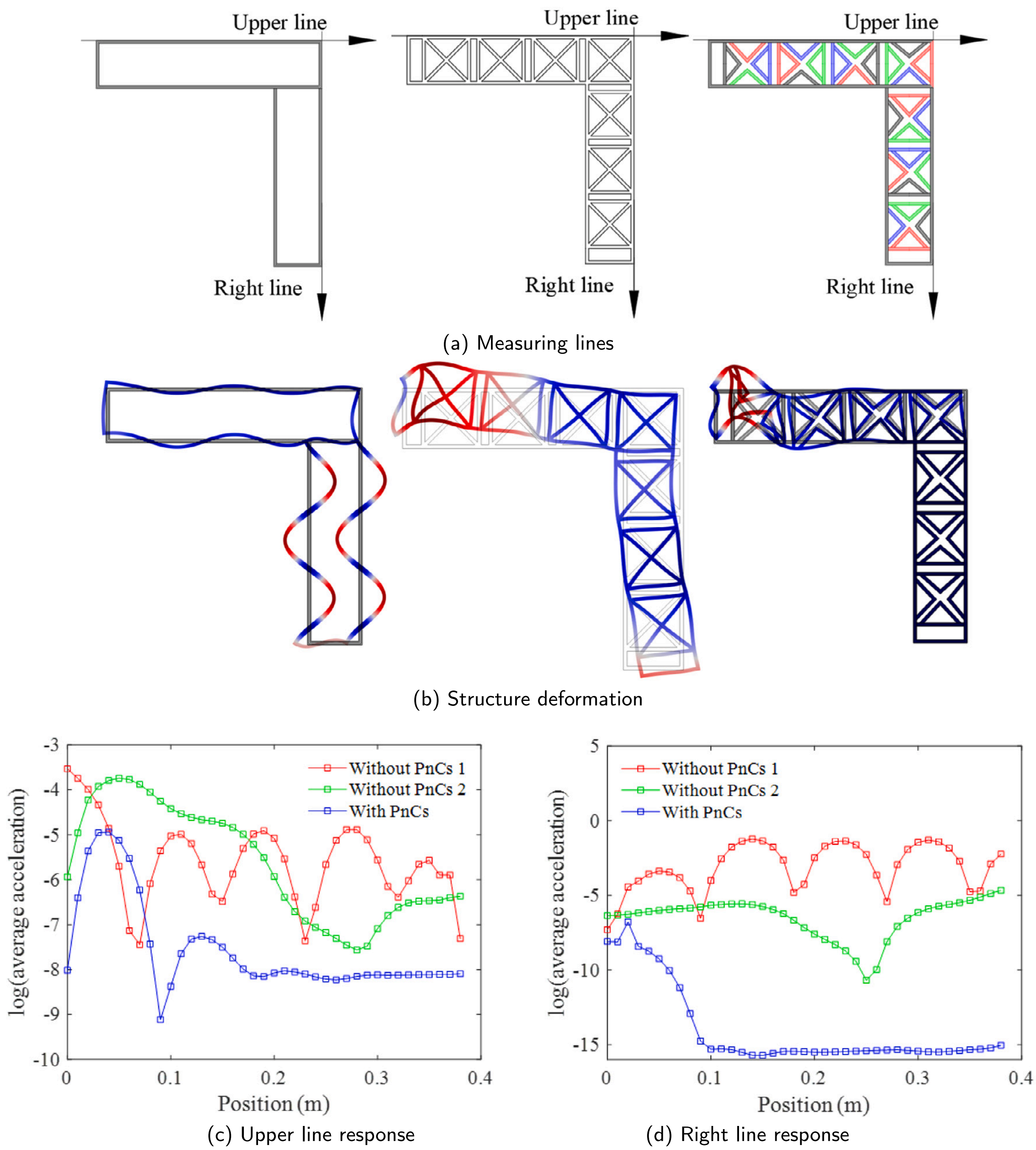


Fig. 13. Structure deformation and acceleration at 1000 Hz.

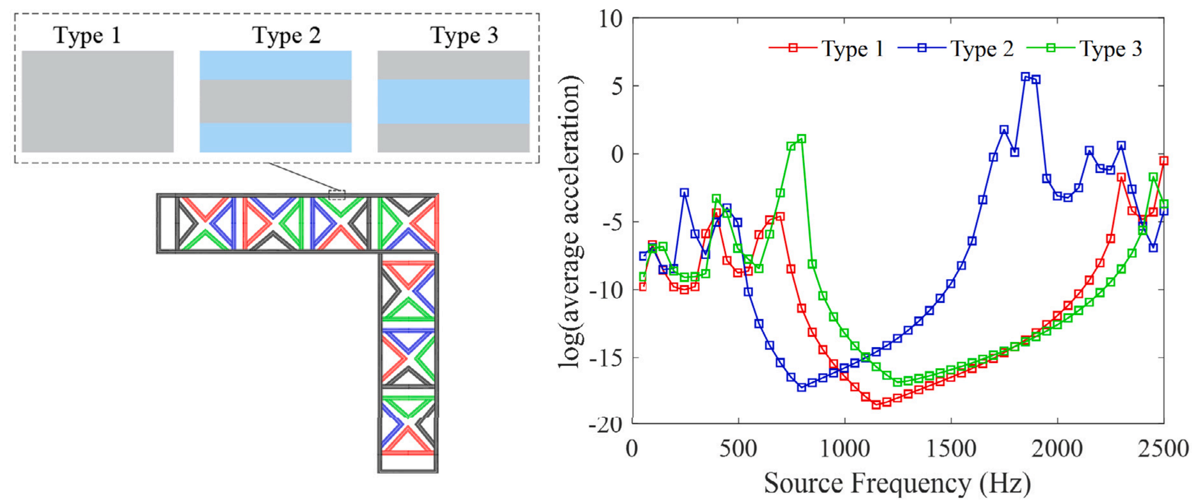


Fig. 14. Structure response with different material configuration(Type 1: Pure Tungsten with a thickness of 5 mm, Type 2:Layer 1 and Layer 3 for Aluminum with a thickness of 1.5 mm, respectively, Type 3: Layer 1 and Layer 3 are Tungsten with a thickness of 1.5 mm, respectively).

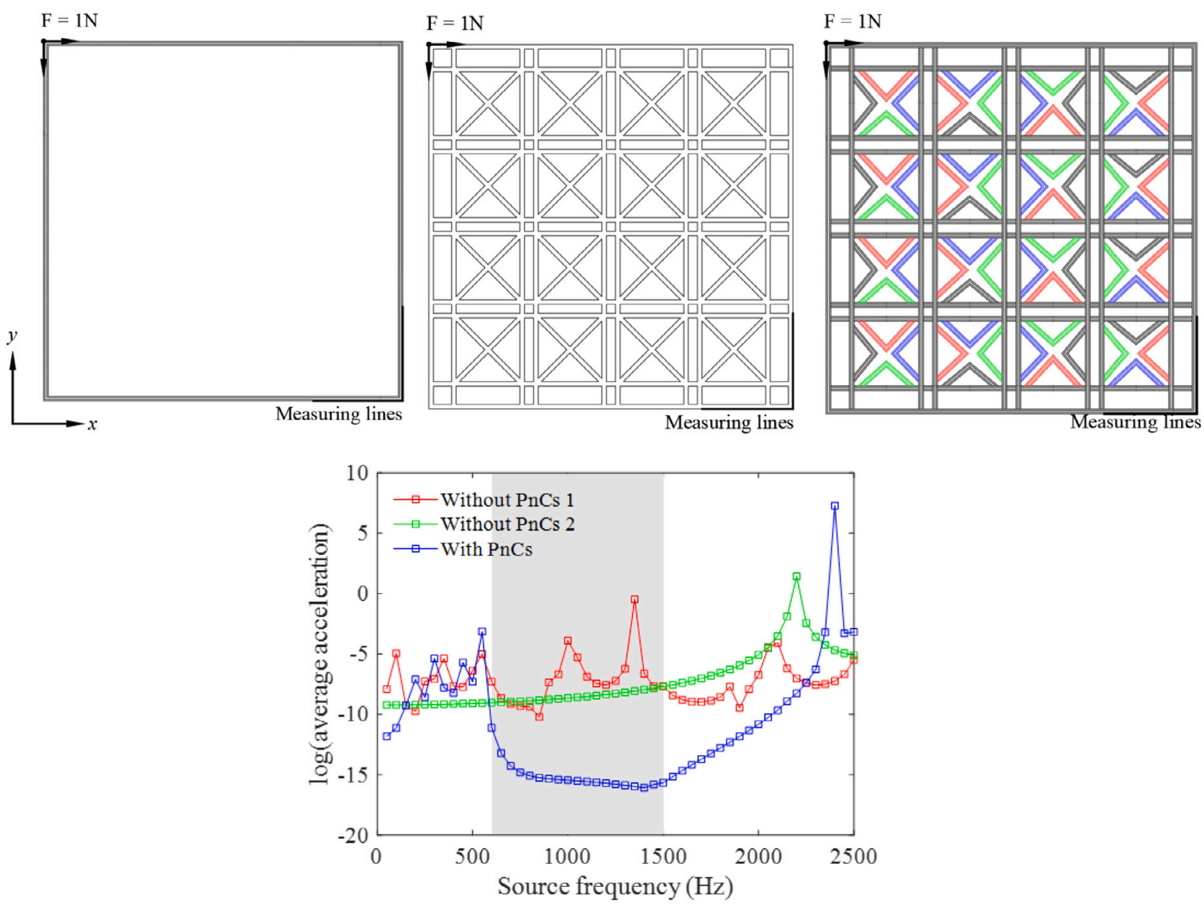


Fig. 15. Structure response under various source frequencies.

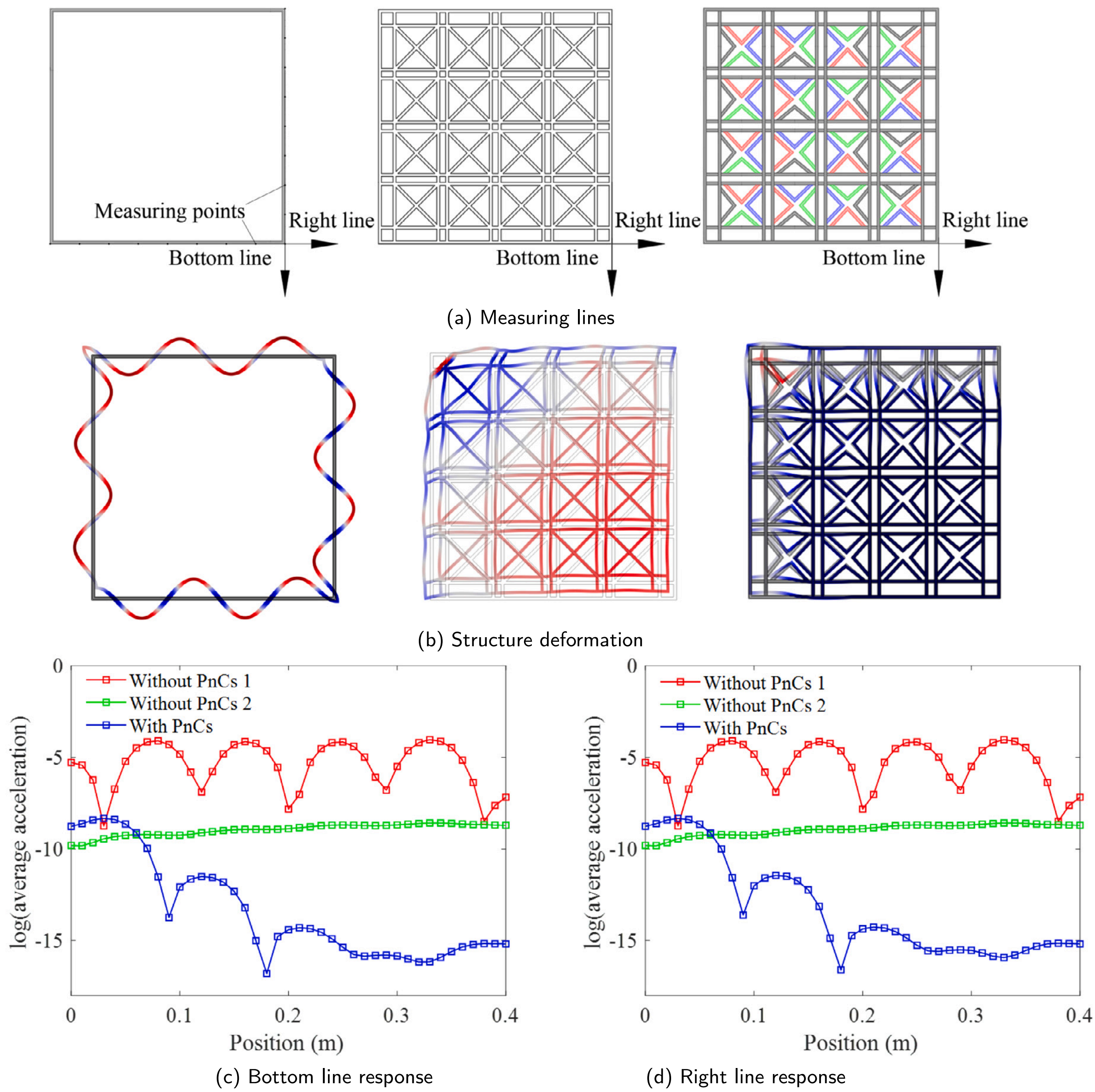


Fig. 16. Structure deformation and acceleration at 1000 Hz.

Appendix

Apply the second Newton law in the vertical direction of the small element, then we can get

$$\tau(x,t) - \tau(x,t) - \frac{\partial \tau(x,t)}{\partial x}dx = \sum_i \rho_i A_i \frac{\partial^2 \Psi(x,t)}{\partial t^2}dx \tag{6}$$

The moment of force of the small element should be in equilibrium, and we can get

$$M(x,t) - M(x,t) - \frac{\partial M(x,t)}{\partial x}dx + \left(\tau(x,t) + \frac{\partial \tau(x,t)}{\partial x}dx\right)dx = 0 \tag{7}$$

As dx times dx is infinitesimal, then Eq. (7) can be rewritten as

$$\tau(x,t) = \frac{\partial M(x,t)}{\partial x} \tag{8}$$

Please note that $M(x,t) = \sum_i E_i I_i \frac{\partial^2 \Psi(x,t)}{\partial x^2}$. Finally, by substituting Eq. (8) into Eq. (6), we can obtain the governing equation of the multi-material Euler–Bernoulli beam.

$$\frac{\partial^4 \Psi(x,t)}{\partial x^4} + \frac{\sum_i \rho_i A_i}{\sum_i E_i I_i}\frac{\partial^2 \Psi(x,t)}{\partial t^2} = 0 \tag{9}$$

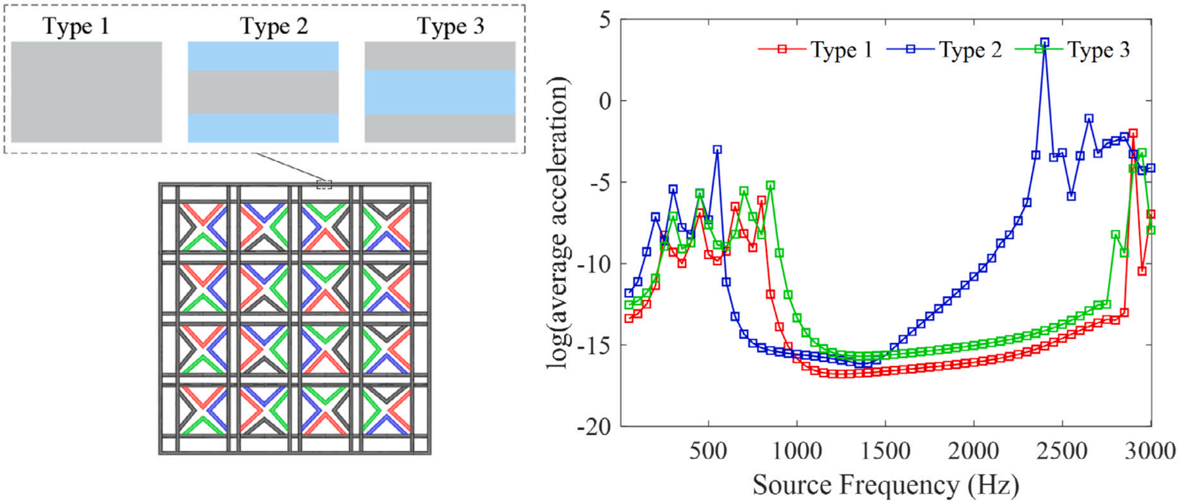


Fig. 17. Structure response with different material configuration(Type 1: Pure Tungsten; Type 2: Layer 1 and Layer 3 for Aluminum with 0.0015 m, respectively; Type 3: Layer 1 and Layer 3 for Tungsten with 0.0015 m, respectively).

References

[1] Song Henan, Shan Xiaobiao, Li Ruirui, Hou Chengwei. Review on the vibration suppression of cantilever beam through piezoelectric materials. Adv Energy Mater 2022;24(11):2200408.

[2] Huang Xingbao, Yang Bintang. Towards novel energy shunt inspired vibration suppression techniques: Principles, designs and applications. Mech Syst Signal Process 2023;182:109496.

[3] Yang Tao, Zhou Shengxi, Fang Shitong, Qin Weiyang, Inman Daniel J. Nonlinear vibration energy harvesting and vibration suppression technologies: Designs, analysis, and applications. Appl Phys Rev 2021;8(3):031317.

[4] He ZC, Xiao X, Li Eric. Design for structural vibration suppression in laminate acoustic metamaterials. Composites B 2017;131:237–52.

[5] An Xiyue, Lai Changliang, He Weiping, Fan Hualin. Three-dimensional chiral meta-plate lattice structures for broad band vibration suppression and sound absorption. Composites B 2021;224:109232.

[6] Yi Guilian, Youn Byeng D. A comprehensive survey on topology optimization of phononic crystals. Struct Multidiscip Optim 2016;54:1315–44.

[7] Li Weibai, Meng Fei, Chen Yafeng, Li Yang fan, Huang Xiaodong. Topology optimization of photonic and phononic crystals and metamaterials: A review. Adv Theory Simul 2019;2(7):1900017.

[8] Kang Se-Hyeon, Choi Myung-Jin, Oh Myung-Hoon, Yoon Min-Ho, Cho Seonho. Isogeometric analysis of lattice structures having compression-twist coupled deformation. J Comput Struct Eng Inst Korea 2021;34(5):287–92.

[9] Assouar Badreddine, Liang Bin, Wu Ying, Li Yong, Cheng Jian-Chun, Jing Yun. Acoustic metasurfaces. Nat Rev Mater 2018;3(12):460–72.

[10] Lu Ming-Hui, Feng Liang, Chen Yan-Feng. Phononic crystals and acoustic metamaterials. Mater Today 2009;12(12):34–42.

[11] Zheng Li-Yang, Wu Ying, Ni Xu, Chen Ze-Guo, Lu Ming-Hui, Chen Yan-Feng. Acoustic cloaking by a near-zero-index phononic crystal. Appl Phys Lett 2014;104(16):161904.

[12] Lv Hangyuan, Tian Xiaoyong, Wang Michael Yu, Li Dichen. Vibration energy harvesting using a phononic crystal with point defect states. Appl Phys Lett 2013;102(3):034103.

[13] Tol Serife, Degertekin Fahrettin Levent, Erturk Alper. Gradient-index phononic crystal lens-based enhancement of elastic wave energy harvesting. Appl Phys Lett 2016;109(6):063902.

[14] Hwan Oh Joo, Jae Kim Yoon, Young Kim Yoon. Wave attenuation and dissipation mechanisms in viscoelastic phononic crystals. 2013.

[15] Panahi Emad, Hosseinkhani Ali, Khansanami Mohammad Farid, Younesian Davood, Ranjbar Mostafa. Novel cross shape phononic crystals with broadband vibration wave attenuation characteristic: Design, modeling and testing. Thin-Walled Struct 2021;163:107665.

[16] Li Qing, Yang Deqing, Mao Xiang. Pressure-resistant cylindrical shell structures comprising graded hybrid zero Poisson's ratio metamaterials with designated band gap characteristics. Mar Struct 2022;84:103221.

[17] Sigmund Ole, Søndergaard Jensen Jakob. Systematic design of phononic band–gap materials and structures by topology optimization. Phil Trans R Soc A 2003;361(1806):1001–19.

[18] Choi Myung-Jin, Oh Myung-Hoon, Cho Seonho, Koo Bonyong. Isogeometric optimal design of Kelvin lattice structures for extremal band gaps. J Comput Struct Eng Inst Korea 2019;32(4):241–7.

[19] Kim Han-Min. Topology optimization of beam splitter for multi-beam forming based on the phase field design method. J Comput Struct Eng Inst Korea 2019;32(3):141–7.

[20] Liu Wenlong, Yoon Gil Ho, Yi Bing, Choi Hyunggyu, Yang Yue. Controlling wave propagation in one-dimensional structures through topology optimization. Comput Struct 2020;241:106368.

[21] Li Weibai, Meng Fei, Li Yang fan, Huang Xiaodong. Topological design of 3D phononic crystals for ultra-wide omnidirectional bandgaps. Struct Multidiscip Optim 2019;60:2405–15.

[22] Shui Xin, Wang Shimin. Investigation on a mechanical vibration absorber with tunable piecewise-linear stiffness. Mech Syst Signal Process 2018;100:330–43.

[23] Pai P Frank, Peng Hao, Jiang Shuyi. Acoustic metamaterial beams based on multi-frequency vibration absorbers. Int J Mech Sci 2014;79:195–205.

[24] Cai Changqi, Zhou Jiaxi, Wu Linchao, Wang Kai, Xu Daolin, Ouyang Huajiang. Design and numerical validation of quasi-zero-stiffness metamaterials for very low-frequency band gaps. Composite Struct 2020;236:111862.

[25] Xu Xianchen, Barnhart Miles V, Li Xiaopeng, Chen Yangyang, Huang Guoliang. Tailoring vibration suppression bands with hierarchical metamaterials containing local resonators. J Sound Vib 2019;442:237–48.

[26] Jin Yang, Shi Yan, Yu Guo-Cai, Wei Guang-Tao, Hu Bin, Wu Lin-Zhi. A multifunctional honeycomb metastructure for vibration suppression. Int J Mech Sci 2020;188:105964.

[27] Wang Ting, Sheng Mei-Ping, Qin Qing-Hua. Multi-flexural band gaps in an Euler–Bernoulli beam with lateral local resonators. Phys Lett A 2016;380(4):525–9.

[28] Miranda Jr. EJP, Nobrega ED, Rodrigues SF, Aranas Jr. C, Dos Santos JMC. Wave attenuation in elastic metamaterial thick plates: Analytical, numerical and experimental investigations. Int J Solids Struct 2020;204:138–52.

[29] Casalotti Arnaldo, El-Borgi Sami, Lacarbonara Walter. Metamaterial beam with embedded nonlinear vibration absorbers. Int J Non-Linear Mech 2018;98:32–42.

[30] Banerjee Arnab. Flexural waves in graded metabeam lattice. Phys Lett A 2021;388:127057.

[31] Zhang Zhaohang, Krushynska AO. Programmable shape-morphing of rose-shaped mechanical metamaterials. APL Mater 2022;10(8).

[32] Dudek KK, Mizzi L, Martínez JA Iglesias, Spaggiari A, Ulliac G, Gatt R, et al. Micro-scale graded mechanical metamaterials exhibiting versatile Poisson's ratio. Compos Struct 2023;319:117151.

[33] Xu Jiawen, Zhang Xin, Yan Ruqiang. Coupled piezoelectric phononic crystal for adaptive broadband wave attenuation by destructive interference. J Appl Mech 2020;87(9):091001.

[34] Fu Pengfei, Lyu Xiaofei, Yang Tianzhi. Wide-band vibration isolation induced by merging acoustic black hole and destructive interference. Available at SSRN 4177913.

[35] Yoon Gil Ho, Shin Myung, Kim Jiwan, Ha Tam Yee. Mechanical metamaterial absorber with destructive interference of transverse vibration. Available at SSRN 4354048.

[36] Sun Jingbo, Liu Lingyun, Dong Guoyan, Zhou Ji. An extremely broad band metamaterial absorber based on destructive interference. Opt Express 2011;19(22):21155–62.

[37] Kanai Tsuneto, Takahashi Eiji J, Nabekawa Yasuo, Midorikawa Katsumi. Destructive interference during high harmonic generation in mixed gases. Phys Rev Lett 2007;98(15):153904.

[38] Polifke Wolfgang, Paschereit Christian Oliver, Döbbeling Klaus. Constructive and destructive interference of acoustic and entropy waves in a premixed combustor with a choked exit. Int J Acoust Vib 2001;6(3):135–46.

[39] Wilson-Rae I, Barton RA, Verbridge SS, Southworth DR, Ilic B, Craighead Harold G, et al. High-Q nanomechanics via destructive interference of elastic waves. Phys Rev Lett 2011;106(4):047205.

[40] Fang Xin, Wen Jihong, Bonello Bernard, Yin Jianfei, Yu Dianlong. Wave propagation in one-dimensional nonlinear acoustic metamaterials. New J Phys 2017;19(5):053007.

[41] Gei Massimiliano. Elastic waves guided by a material interface. Eur J Mech A Solids 2008;27(3):328–45.

[42] Dequand Sylvie, Hulshoff Steven J, Aurégan Yves, Huijnen J, ter Riet R, van Lier LJ, et al. Acoustics of 90 degree sharp bends. Part I: Low-frequency acoustical response. Acta Acust United Acustica 2003;89(6):1025–37.