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# Multi-material topology optimization of phononic crystal considering isotropic/anisotropic materials

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ABSTRACT

Multi-material phononic crystals hold promise for manipulating elastic wave propagation, enhancing the rigidity of the host structure, and realizing multifunctionality, including electric conduction, sound insulation, and heat diffusion. This paper presents a multi-material topology optimization pipeline for phononic crystal design, incorporating both isotropic and anisotropic materials. First, the dispersion theory for elastic wave propagation in periodic structures is presented. Then a novel interpolation function is proposed for multi-material topology optimization by using a variant of the projection operator. Finally, both isotropic and anisotropic materials are utilized to demonstrate the effectiveness of the proposed method for multi-material phononic crystal design when compared with SIMP-based structures. The numerical analysis indicates that the proposed method performs well in optimizing the phononic structure with metal composite materials.

## 1. Introduction

Continuum structures including beams and plates are widely used in the field of civil engineering, mechanical products, and aerospace equipment. Elastic wave manipulation is one of the significant and challenging tasks to ensure the safety and functionality of the engineering structures. Phononic crystals, periodically arranged artificial materials/structures, show extraordinary performance on forbidding elastic wave propagation in band gaps which offers the possibility of wave manipulation in practical engineering applications [1–4]. The multi-material phononic structure offers advantages such as high strength, lightweight construction, and cost-effectiveness, while satisfying the requirements of elastic wave manipulation. [5–7]. To further contribute to this topic, multiple materials-based topology optimization method for phononic band gap design with both isotropic and anisotropic materials is studied in this paper.

Topology optimization technologies including solid isotropic material with penalization (SIMP) [8,9], Level-Set method (LSM) [10,11], bidirectional evolutionary structural optimization (BESO) [12,13], and moving morphable component (MMC) [14,15], offer an effective and efficient way for structure design. In recent years, topology optimization for phononic crystal design has drawn tremendous attention. Relevant literature regarding phononic crystal design can be categorized into non-gradient and gradient-based methods. Non-gradient-based phononic structure optimization mainly relies on random processes such as genetic algorithms, which do not need any gradient information for structure design [16–18]. Dong et al. [19] utilized a genetic algorithm with the adaptive fuzzy fitness granulation for maximizing the relative bandgap width of the asymmetrical phononic structure, which improved the computation efficiency for the optimization procedure. With the help of a genetic algorithm and plane wave expansion method, Han et al. optimized the band gap of the phononic crystal with three phases in both the out- and in-plane wave modes, the results indicate the volume fraction and symmetry are significant for optimal design in [20]. Combining adaptive genetic algorithm, Xie proposed an improved fast plane wave expansion method for 2d phononic crystal with a symmetric square lattice microstructure, in which the computational efficiency has been significantly improved in [21]. The non-gradient methods provide an easy but effective way for the structure design. However, the computation efficiency highly depends on optimization objectives and variables, and the computational time is not acceptable to some degree when the number of design variables is large.

The gradient-based method as an efficient technology for solving optimization problems with few numbers of evaluation functions has been widely applied in structural compliance, acoustic metamaterials, and photonic/phononic crystals design [22–25]. Sigmund et al. [26] pro-

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posed a solid isotropic material with penalization pipeline for band gap maximization, structure response minimization, and waveguiding. Similarly, Wang et al. [27] utilized a bidirectional evolutionary structural optimization algorithm for phononic crystals design with an ultrawide band gap. Liu et al. [28] presented a multi-functional topology optimization framework for phononic crystal design, which realized elastic wave manipulation in a one-dimensional structure. In [29], a level set-based topology optimization procedure with an interface-enriched generalized finite element method is presented for designing the band structure, which improves the boundary of the phononic crystals compared with the density-based approaches. To design phononic crystals with prescribed band gaps, Wu et al. [30] introduced a novel topology optimization method with ipsilateral frequency constraint based on a modified Heaviside function and robust formulation, which is flexible in practical engineering applications.

Using multiple materials for topology optimization offers more freedom in structural design, which can further improve the stability, weight and other physical performance [31–35]. With the advancements in multi-material processing and manufacturing technologies, the exploration of multi-material topology optimization has emerged as a focal point in academic discourse. In [36], Bendsoe et al. first extended the SIMP interpolation function model for multi-material topology optimization. Discrete material optimization, a simple but effective interpolation function was proposed for multi-material topology optimization in [37]. Bing et al. presented a unified material interpolation function for multi-material topology optimization using the p-norm penalization function [31]. Based on the multi-material topology optimization framework, a few researchers attempted to design phononic crystals with multiple materials for manipulating elastic wave propagation. Combining compactly supported radial basis functions and Heaviside functions, Chen et al. proposed a single variable-based multi-material topology optimization method for designing the band gap of phononic structure in [5]. Xu et al. systematically investigated a two-stage multi-objective topology optimization pipeline for phononic crystal design with multiple phases of microstructure. In [38], Zhang et al. presented a systematic topology optimization method of multi-material metamaterial with an interpolation scheme of discrete material optimization, which realized Bragg scattering and local resonance mechanism-based metamaterial design with ultra-wide low-frequency band gaps. In literature [39], Yi et al. comprehensively reviews the progress on topology optimization of phononic crystal, the basic theory and method of phononic crystal design and optimization are detailed, also the functionality of the phononic crystals is introduced. However, to the best of our knowledge, multi-material topology optimization for phononic crystals is still challenging and immature, especially the interpolation model for multiple-material mapping.

Furthermore, the layout of the topology optimization-based phononic crystal design is fairly complicated to manufacture by conventional technology, thanks to the advancement of additive manufacturing, the fabrication of the optimized phononic crystal becomes possible. However, layer-by-layer processing leads to different material properties along the fabrication directions. In addition, internal structure of polycrystalline materials such as composite metals and random imperfection of materials impact the structural property as well [40–42]. Therefore, to further contribute to this topic, a novel exponential interpolation function for multi-material topology optimization-based band gap optimization is presented to design phononic crystals, and both the isotropic and anisotropic materials are introduced to evaluate the wave dispersion phenomenon. The main ***contributions*** of this paper are summarized as follows:

1. A new exponential interpolation function is presented for multi-material topology optimization, which can effectively realize the structural design of phononic crystals.
2. Both isotropic and anisotropic materials are being investigated to maximize the bandgap of the phononic structure for manipulating elastic wave propagation, and to explore the effects of anisotropic factors on the bandgap.
3. Compared with the SIMP-based optimization method for phononic crystal design, the proposed method has a wider band gap to suppress the elastic wave in the phononic crystal structure.

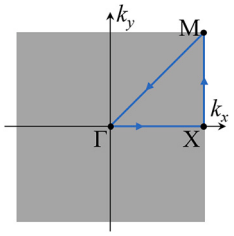



**Fig. 1.** Irreducible Brillouin zone.

The remainder of this paper is organized as follows: In Section 2, the dispersion theory for wave propagation in periodic structure is described. Section 3 detailed the multi-materials topology optimization for the phononic structure. Numerical examples including isotropic and anisotropic materials are utilized to demonstrate the performance of the proposed method in Section 4. Finally, the conclusion and future work are presented in Section 5.

## 2. Dispersion theory for periodic structures

Before using the topology optimization method to design the phononic crystal for elastic wave manipulation, the dispersive relationship should be derived to analyze the band gap of the phononic structure. The governing equation for wave propagation in the periodic structure can be formulated as:

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2} = (\lambda+2\mu)\nabla\nabla\cdot\mathbf{u} - \mu\nabla\times\nabla\times\mathbf{u} \tag{1}$$

where u and $\rho$ are the displacement vector and material density, respectively, parameters $\lambda$ and $\mu$ are Lame coefficients related to material property. Based on the periodicity and symmetry of the phononic structure, the wave propagation in the periodic structure can be analyzed in a periodic unit cell. According to Floquet-Bloch theory, the displacement vector can be described as follows:

$$\mathbf{u}(\mathbf{r},\mathbf{k}) = \mathbf{u}_{\mathbf{k}}(\mathbf{r})\, e^{i\mathbf{k}\cdot\mathbf{r}} \tag{2}$$

where r and k denote the position and wave vector, respectively. An eigenvalue problem can be formulated in Eq. (3) by combining Eqs. (1), (2):

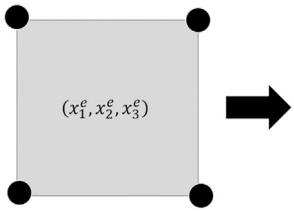
$$\mathbf{K}\mathbf{u} = \omega^2\mathbf{M}\mathbf{u} \tag{3}$$

where $\omega^2$ is the eigenvalue of the system, and **K** and **M** denote global stiffness and mass matrices, respectively. Based on the above analysis, the dispersive curve of the elastic wave propagating within the phononic crystal can be depicted by sweeping the wave vector along the edge of the irreducible Brillouin zone shown in Fig. 1.

In this paper, we take a one-dimensional phononic structure as an example for manipulating elastic waves. A combination of $k_x = [-\pi, \pi]$ and $k_y = 0$ is utilized to analyze the elastic wave propagation in $\Gamma X$ direction, and the finite element method is used to solve the eigenfrequency problem shown in Eq. (3).

## 3. Multi-material topology optimization for phononic crystal design

In this section, an exponential density interpolation function is presented for multi-material, and filtering and projection operators are utilized to deal with checkboard and gray elements problems, then the




| $(x_1^e, x_2^e, x_3^e)$ | $\mathbf{K}(x_1^e, x_2^e, x_3^e)$ | $\mathbf{K}(x_1^e, x_2^e, x_3^e)$ |
|---|---|---|
| (0,0,0) | $\mathbf{K}_{\text{void}}$ | $\mathbf{M}_{\text{void}}$ |
| (1,0,0) | $\mathbf{K}_1$ | $\mathbf{M}_1$ |
| (1,1,0) | $\mathbf{K}_2$ | $\mathbf{M}_2$ |
| (1,1,1) | $\mathbf{K}_3$ | $\mathbf{M}_3$ |

**Fig. 2.** Interpolation function for multiple materials.

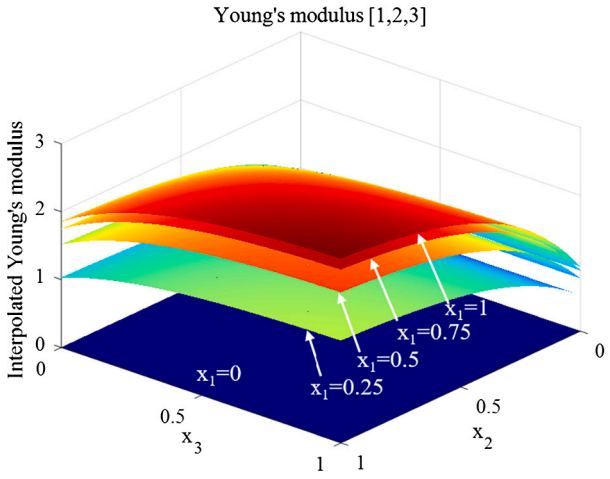



**Fig. 3.** Interpolated Young's modulus with proposed method.

objective function is formulated for phononic crystal design based on multi-material topology optimization.

### 3.1. Interpolation function

For multi-material topology optimization-based phononic structure design, how to construct the mapping relationship between the design variable and material distribution is significant in the optimization process. A new interpolation function for multi-material topology optimization is constructed as Eq. (4):

$$\phi_i^e = \left(1 - e^{-\alpha x_1^e}\right)\left(e^{-\alpha x_{i+1\neq NM+1}^e}\right)\left[\prod_{j=2}^{i}\left(1 - e^{-\alpha x_j^e}\right)\right] \tag{4}$$

where $x$ and $\phi$ indicate design variable and the interpolated one, respectively, and subscript i and superscript e are the index of materials and elements, respective, and NM indicates the total number of materials. To better understand the interpolation function, an example with three materials is presented in Eq. (5) and illustrated in Fig. 2, in which $x_1$, $x_2$, and $x_3$ denote design variables for materials 1, 2, and 3, respectively. Parameter $\alpha$ controls the sharpness of the mapping function which plays a similar role with the projection function.

$$\begin{aligned}
\phi_1^e &= (1 - e^{-\alpha x_1^e})e^{-\alpha x_2^e} \\
\phi_2^e &= (1 - e^{-\alpha x_1^e})(1 - e^{-\alpha x_2^e})e^{-\alpha x_3^e} \\
\phi_3^e &= (1 - e^{-\alpha x_1^e})(1 - e^{-\alpha x_2^e})(1 - e^{-\alpha x_3^e})
\end{aligned} \tag{5}$$

Fig. 3 shows an interpolated Young's modulus with the proposed mapping function, where the Young's modulus for three materials are 1, 2, and 3, respectively. It is observed that the design variables can be effectively mapped into multiple material physical fields.

### 3.2. Optimization

In this paper, we focus on multi-material topology optimization-based phononic structure design. The aim is to maximize the width of the band gap by iterative optimization. Based on the aforementioned material interpolation model, the mathematical formulation for phononic structure design can be constructed as follows:

$$\begin{aligned}
&\underset{\mathbf{x}}{\text{maximize}} : \omega_{j+1}(\mathbf{k}) - \omega_j(\mathbf{k}) \\
&\text{subject to} : \begin{cases} V(\mathbf{x})/V_0 \le f_0 \\ \mathbf{Ku} = \omega^2\mathbf{Mu} \\ 0 \le x_i^e \le 1 \end{cases}
\end{aligned} \tag{6}$$

where $\omega_j$ and $\omega_{j+1}$ are the $j^{th}$ and $j+1^{th}$ eigen frequencies, $\mathbf{x}$ is the design variable which has the following form shown in Eq. (7), NE indicates the number of the elements. $V(x)$, $V_0$ and $f_0$ indicate the volume of the optimized structure, design domain, and the prescribed volume fraction, respectively.

$$\mathbf{x} = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_{NM}^1 \\ x_1^2 & x_2^2 & \cdots & x_{NM}^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{NE} & x_2^{NE} & \cdots & x_{NM}^{NE} \end{bmatrix} \tag{7}$$

In this paper, we consider both the isotropic and anisotropic materials for phononic crystal design. A uniform form of the elastic matrix for both homogenization and non-homogenization material is shown in Eq. (8), where $E_x = \gamma_1 E, E_y = \gamma_2 E$ are Young's modulus along x and y directions, G and $\nu$ are shear modulus and Poisson's ratio, respectively, and $\gamma_1$ and $\gamma_2$ are related material properties.

$$D_i = \begin{bmatrix} \frac{E_x^i * E_x^i}{E_x^i - E_y^i * \nu^2} & \frac{E_x^i * E_y^i * \upsilon_{12}^i}{E_x^i - E_y^i * \nu^2} & 0 \\ \frac{E_x^i * E_y^i * \upsilon_{12}}{E_x^i - E_y^i * \nu^2} & \frac{E_x^i * E_y^i}{E_x^i - E_y^i * \nu^2} & 0 \\ 0 & 0 & G \end{bmatrix} \tag{8}$$

### 3.3. Sensitivity analysis

Based on the stated mathematical formulation, we briefly introduce the sensitivity analysis which is used to update the design variables.

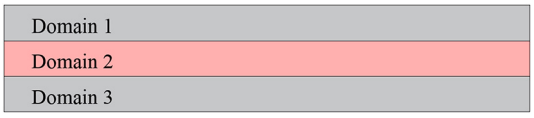



**Fig. 4.** Design domain for phononic crystal design.

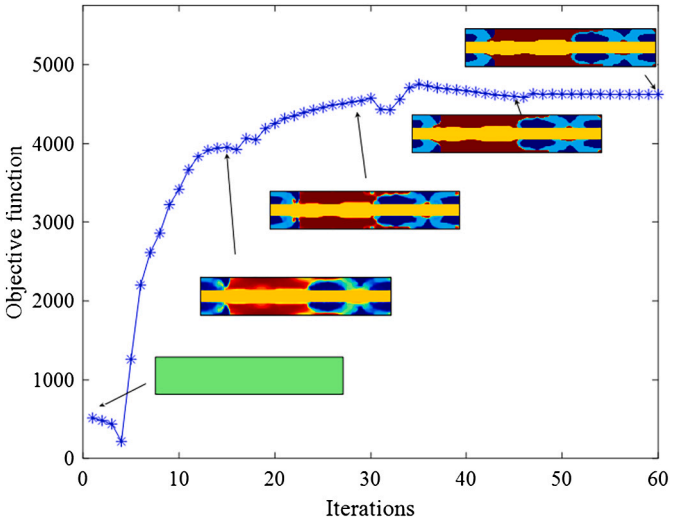
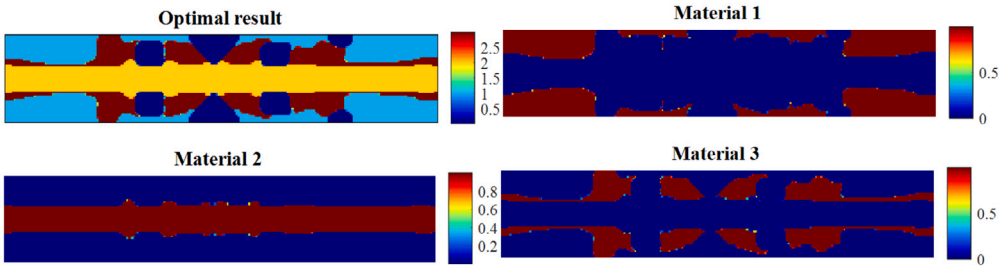
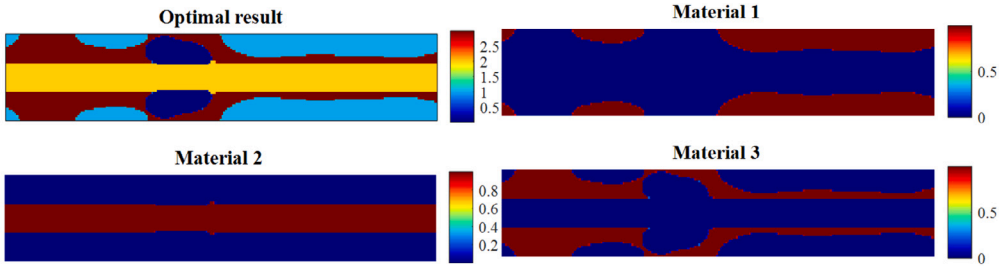



**Fig. 5.** Iteration history of the phononic crystal.

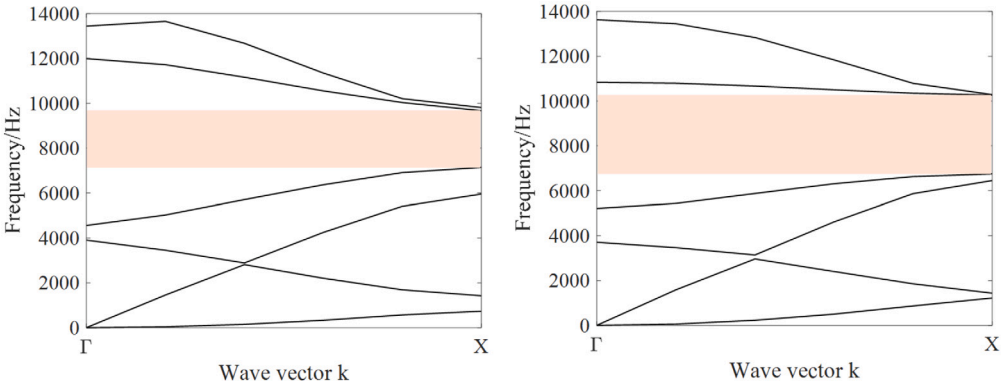



(a) SIMP



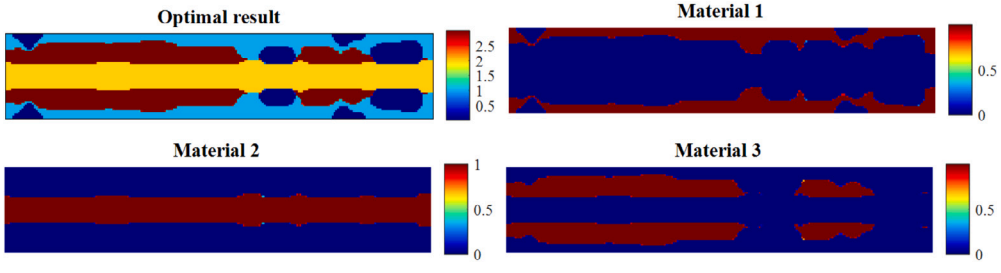

(b) Proposed

**Fig. 6.** Materials distribution for phononic structure.

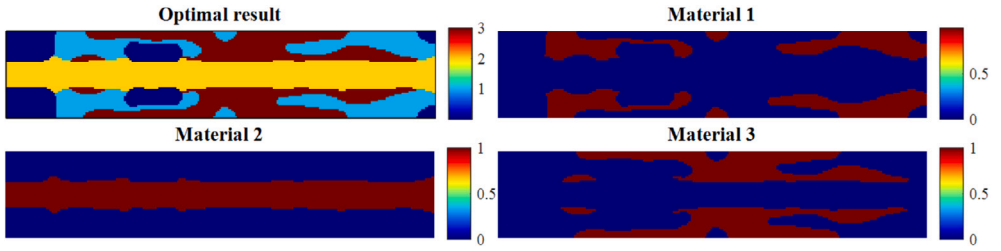



(a) SIMP (b) Proposed

**Fig. 7.** Dispersion curves of the phononic structures.




(a) SIMP




(b) Proposed

**Fig. 8.** Materials distribution for phononic structures.

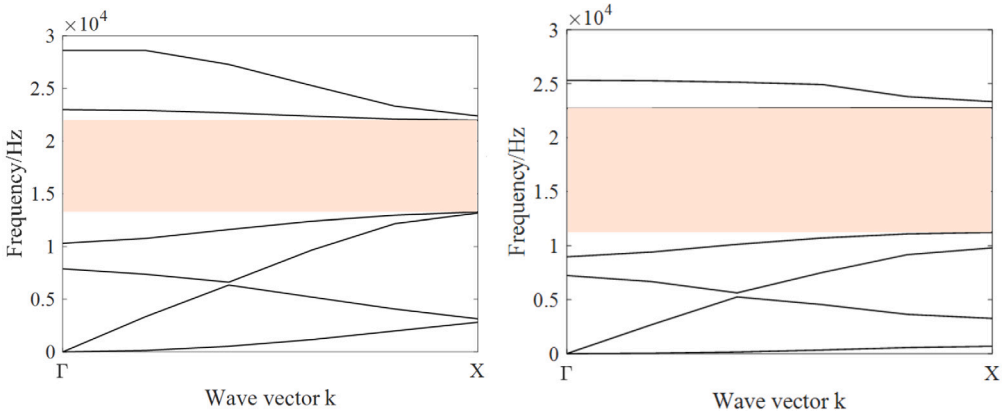



**Fig. 9.** Dispersion curves of the phononic structures.

The sensitivity of the eigenfrequencies concerning material densities is derived in Eq. (9):

$$\frac{\partial\omega(\mathbf{k})}{\partial\phi_i^e}=\frac{1}{2\omega}\mathbf{u}^{\mathbf{T}}\left(\frac{\partial\mathbf{K}}{\partial\phi_i^e}-\omega^2\frac{\partial\mathbf{M}}{\partial\phi_i^e}\right)\mathbf{u} \tag{9}$$

where $\phi_i$ is the i-th interpolated material density, and $\mathbf{K}$ and $\mathbf{M}$ are global stiffness and mass matrices, respectively. The $pq$ penalty strategy is utilized for eigenvalue problem analysis in this paper, thus, $\mathbf{K}$ and $\mathbf{M}$ have the following form shown in Eq. (10), which are assembled from the element stiffness matrices $\mathbf{K^e}$ and mass matrices $\mathbf{M^e}$, respectively.

$$\begin{aligned}\mathbf{K}&=A_{e=1}^{NE}\sum_{i=1}^{NM}\mathbf{K^e}\cdot\left(\phi_i^e\right)^p\\ \mathbf{M}&=A_{e=1}^{NE}\sum_{i=1}^{NM}\mathbf{M^e}\cdot\left(\phi_i^e\right)^q\end{aligned} \tag{10}$$

The sensitivity of the stiffness and mass matrices with respect to interpolated density can be constructed as:

$$\begin{aligned}\frac{\partial\mathbf{K}}{\partial\phi_i^e}&=pA_{e=1}^{NE}\sum_{i=1}^{NM}\mathbf{K^e}\cdot\left(\phi_i^e\right)^{p-1}\\ \frac{\partial\mathbf{M}}{\partial\phi_i^e}&=qA_{e=1}^{NE}\sum_{i=1}^{NM}\mathbf{M^e}\cdot\left(\phi_i^e\right)^{q-1}\end{aligned} \tag{11}$$

The sensitivity of the interpolated density with respect to design variables is formulated as follows:

$$\begin{aligned}\frac{\partial\phi_1^e}{\partial x_1^e}&=\alpha e^{-\alpha x_1^e}e^{-\alpha x_2^e}\\ \frac{\partial\phi_1^e}{\partial x_2^e}&=-\alpha\left(1-e^{-\alpha x_1^e}\right)e^{-\alpha x_2^e}\\ \frac{\partial\phi_1^e}{\partial x_3^e}&=0\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{\partial\phi_2^e}{\partial x_1^e}&=\alpha e^{-\alpha x_1^e}\left(1-e^{-\alpha x_2^e}\right)e^{-\alpha x_3^e}\\ \frac{\partial\phi_2^e}{\partial x_2^e}&=-\alpha\left(1-e^{-\alpha x_1^e}\right)e^{-\alpha x_2^e}e^{-\alpha x_3^e}\\ \frac{\partial\phi_2}{\partial x_3}&=-\alpha\left(1-e^{-\alpha x_1^e}\right)\left(1-e^{-\alpha x_2^e}\right)e^{-\alpha x_2^e}\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial\phi_3^e}{\partial x_1^e}&=\alpha e^{-\alpha x_1^e}\left(1-e^{-\alpha x_2}\right)\left(1-e^{-\alpha x_3^e}\right)\\ \frac{\partial\phi_3^e}{\partial x_2^e}&=\alpha e^{-\alpha x_2^e}\left(1-e^{-\alpha x_1}\right)\left(1-e^{-\alpha x_3^e}\right)\\ \frac{\partial\phi_3^e}{\partial x_3^e}&=\alpha e^{-\alpha x_3^e}\left(1-e^{-\alpha x_1}\right)\left(1-e^{-\alpha x_2^e}\right)\end{aligned} \tag{14}$$

## 4. Numerical analysis and discussion

In this section, numerical analysis is conducted to verify the effectiveness of the proposed method for phononic crystals. All the examples are implemented in MATLAB 2022 and the Method of Moving Asymptotes (MMA) is utilized as a solver for topology optimization [43].

The overall domain for phononic crystal is given in Fig. 4, where domain 2 is a non-design domain that is initially applied with $2^{nd}$ material, and domains 1 and 3 are the designed areas that are applied with $1^{st}$ and $3^{rd}$ materials, of which the initialized densities of three materials are 0.5, 1, and 0.5, respectively. The width and height of the unit cell are 0.03 m and 0.15 m, respectively. The unit cell is discretized into $240\times48$ mesh grids, and the volume constraint of each material is 7/24. The filtering radius is the double mesh size, and the total iteration number is 60. The penalty factor of interpolation function for SIMP method is set to 3, and $\alpha$ for the proposed interpolation model is set to 3 as well, and p and q for eigenfrequency problem are set to 3 and 4, respectively.

Fig. 5 shows an example of the iteration history of the phononic crystal optimization, the material properties follow the example of Section 4.3. It can be found that the initialized design domain is applied with uniform density, and the overall structure of the phononic crystal gradually approaches a clear pattern after a couple of iteration steps, where the red color is material 1, yellow one indicates materials 2, and the sky blue one is the material 3. It is clear that the proposed method is effective on phononic crystal design from the iteration history shown in Fig. 5.

### 4.1. Evaluation on isotropic materials

This case study is to evaluate the proposed method for phononic crystal optimization with isotropic materials. The imaginary material properties for phononic structure design are as follows: $E_1=0.5\text{e}7$, $E_2=1\text{e}7, E_3=0.8\text{e}7$, the density $\rho$ and Poisson's ratio $\nu$ for three materials are set to 1, and 0.3, respectively.

To demonstrate the effectiveness and the advancement of the proposed method, SIMP-based multi-material interpolation model is introduced for comparison. Fig. 6 shows the isotropic material distribution of the phononic structures via SIMP and the proposed method, and Fig. 7 presents the dispersion relations of the optimized phononic crystals, which shows that both the SIMP and proposed method can enlarge the bandgap of the phononic structures, specifically, the proposed method is of better performance on widening bandgap when compared with SIMP method, and the boundary of each component is more clearly as well.

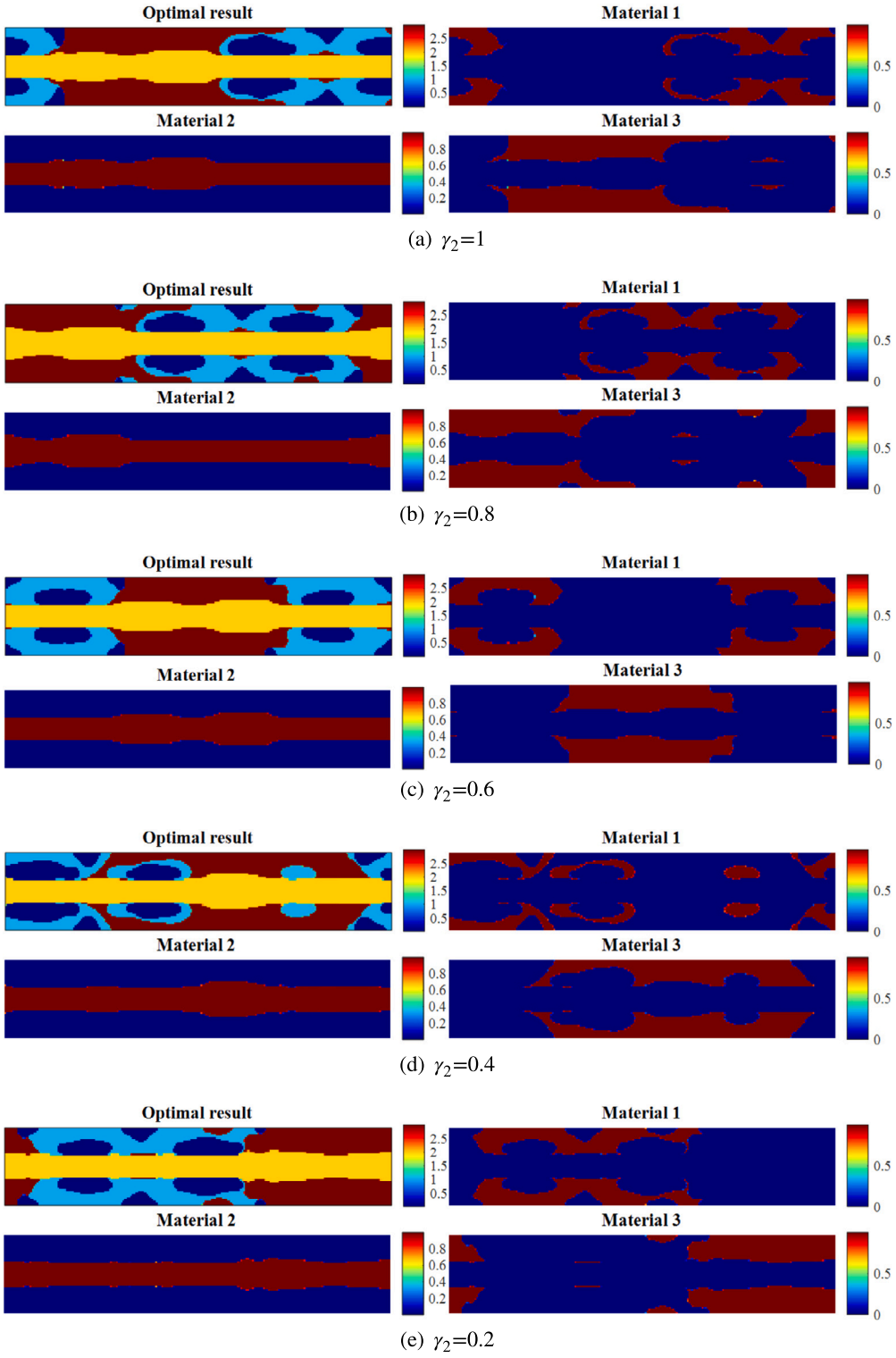



**Fig. 10.** Material distribution of various anisotropic phononic structures, in which $\gamma_1$ is 1.

### *4.2. Evaluation on anisotropic materials*

The second example is conducted to verify the proposed method in phononic structure design with anisotropic materials for materials 1 and 3. The material properties for phononic structure design are as follows: $E_1 = 5e7$, $E_2 = 1e7, E_3 = 10e7$, $\gamma_1 = 0.8$, $\gamma_2 = 1$, the density $\rho$ and Poisson's ratio $\nu$ for three materials are set to 1, and 0.3, respectively.

Fig. 8 gives the materials layout of the optimized phononic structures with anisotropic materials. It can be found that both the SIMP and the proposed method can effectively optimize the structures. Fig. 9 shows the dispersion curves of the optimized phononic crystals, which indicates that both the SIMP and the proposed method can effectively enlarge the bandgap of the phononic structures. Moreover, the bandgap of the optimized structures via the proposed method is larger than the SIMP one.

Overall, both the numerical optimizations on isotropic and anisotropic materials demonstrate the effectiveness of the proposed method in widening the bandgap of the phononic structures. Addition-

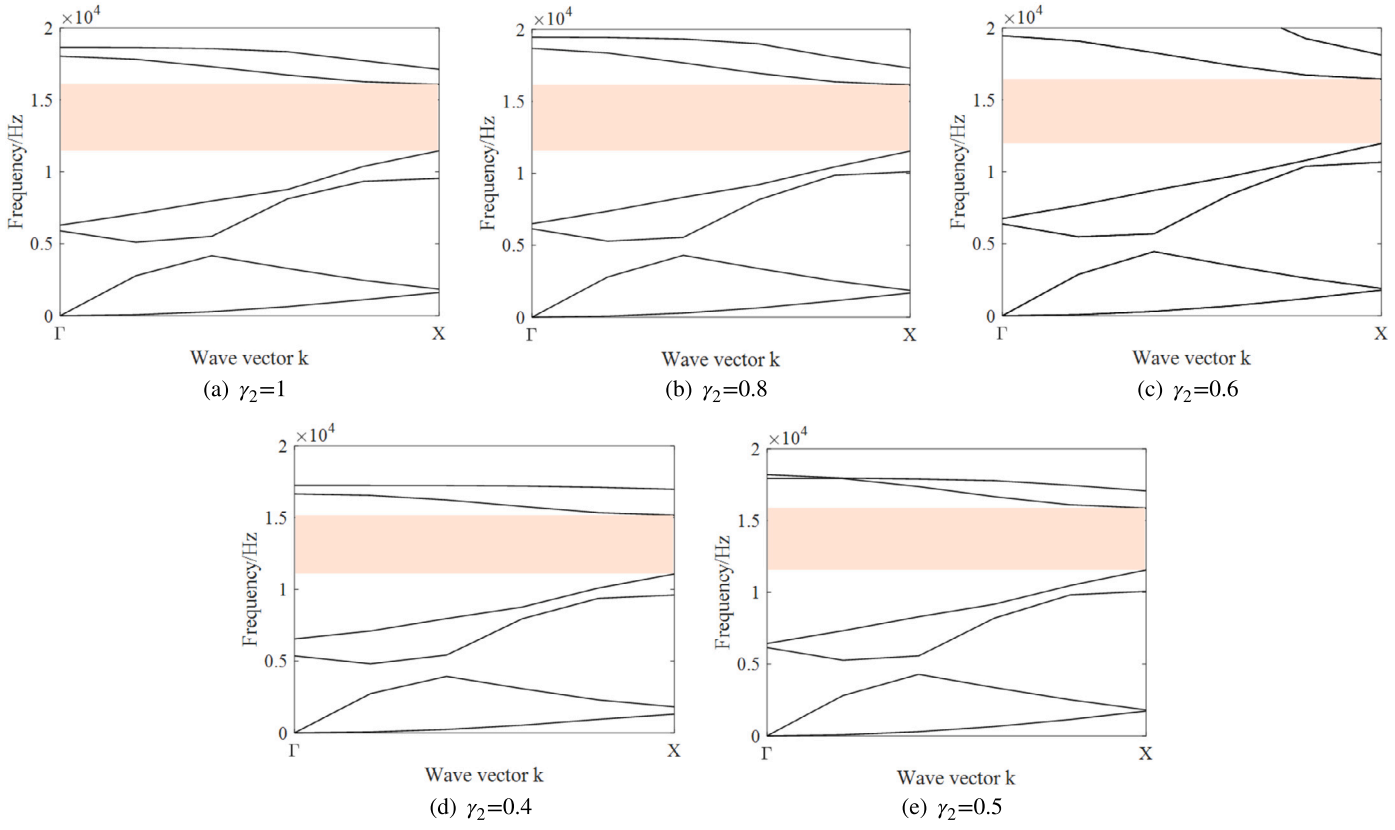



**Fig. 11.** Band gap of the phononic crystals with anisotropic materials.

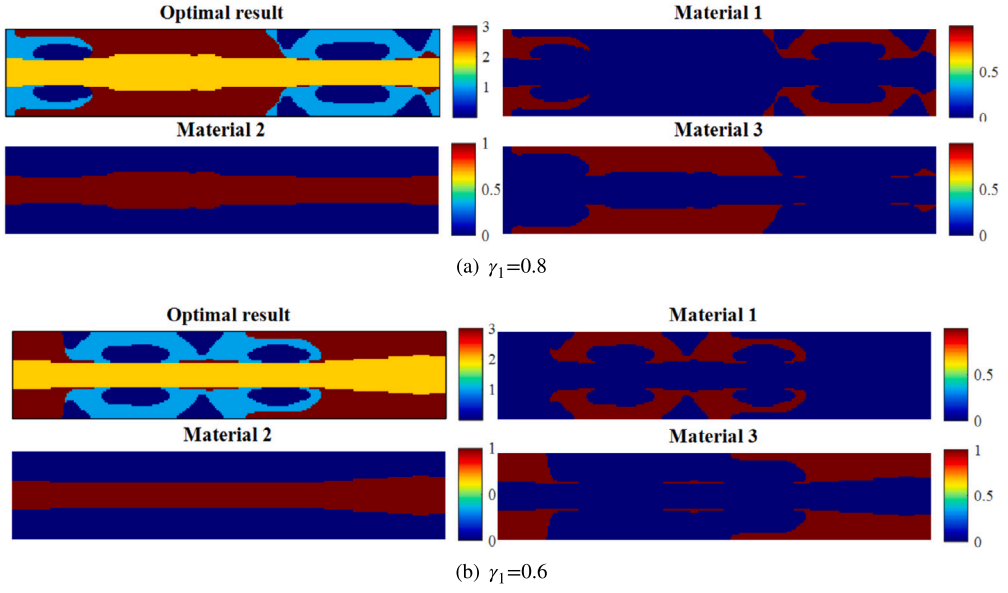



**Fig. 12.** Phononic crystals with anisotropic materials.

ally, the proposed method is of better performance on phononic crystal design from the point of enlarging the bandgap.

### 4.3. Practicability on metal composite

In this subsection, the real materials including aluminum, steel, and copper are introduced to demonstrate the performance of the proposed method on phononic crystals optimization, of which the Young's modulus of three materials are 70GPa, 200GPa, 120GPa, respectively, and the corresponding densities are $2700kg/m^3, 7850kg/m^3, 8900kg/m^3$, respectively. Different anisotropic factors are utilized to analyze the bandgap of the optimized phononic structures.

The first case sets $\gamma_1$ to 1, and the anisotropic factors $\gamma_2$ for optimization are 1, 0.8, 0.6, 0.4, and 0.2, respectively. Fig. 10 shows the optimized phononic structures. We can find that the proposed method is of good performance on composite metals-based phononic design, and the optimized structures have clear boundaries. The overall patterns of the optimized phononic crystal are similar, and the details of the phononic structure are different in some parts. Fig. 11 gives the dispersion relationship of the optimized structures, it can be found that the

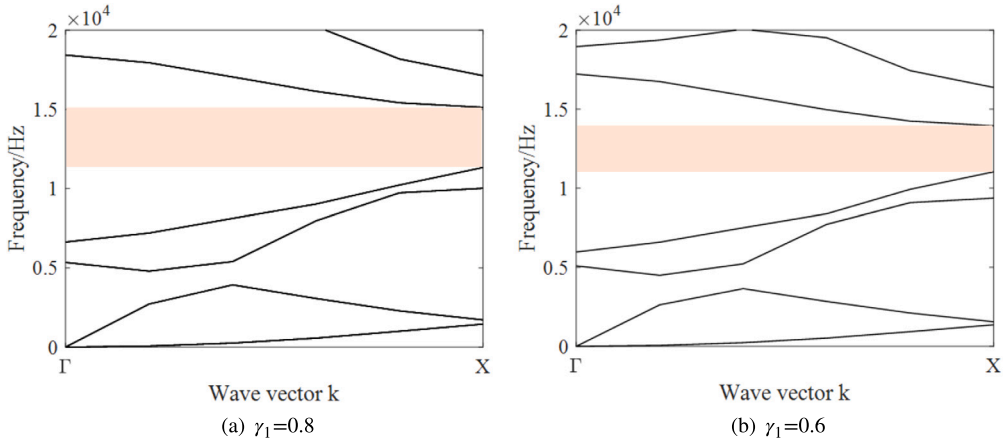



**Fig. 13.** Band gap of phononic crystals with anisotropic materials.

widths of the band gap are almost same. Thus, we can conclude that the anisotropic factor in the Y direction does not have significant effects on the elastic wave propagation along the X direction.

The second case aims to investigate the effect of the different anisotropic factors in the X axis on elastic wave propagation in the X direction. We set $\gamma_2$ to 1, and anisotropic factors $\gamma_1$ are 0.8, and 0.6, respectively. Compared with the phononic structure using isotropic materials shown in Fig. 10(a) and Fig. 11(a), the band gap distribution is different from the first case. It is observed that the width of the band gap turns narrow as the anisotropic factors decrease. Overall, it can be concluded that the proposed method performs well on phononic crystal design with multiple materials in both isotropic and anisotropic materials. (See Figs. 12 and 13.)

## 5. Conclusion and future work

This paper presents a topology optimization framework for phononic crystal design with multiple materials. By using a variant of the projection operator, a novel interpolation function is presented for describing the multi-material fields. Numerical examples are implemented to demonstrate the effectiveness of the proposed pipeline for phononic structure design with multiple materials. The numerical analysis, including both isotropic and anisotropic materials, indicates that the proposed method performs well on phononic crystal design when compared with conventional SIMP-based multi-material frameworks. The metal composite, aluminum-steel-copper, is utilized to verify the effectiveness of the proposed method in practical applications.

The proposed phononic structure optimization pipeline, using multiple isotropic/anisotropic materials, can be applied to mechanical product design and railway infrastructure construction. To exploit the wave dispersion phenomenon in microstructure, multi-scale phononic structure design with multi-materials can be further studied in future work. Additionally, a multi-material-based topology optimization method to design metastructure for wave focusing and tapping can be further studied in the future.

## CRediT authorship contribution statement

**Long Liu:** Writing – original draft, Methodology. **Ji Wan Kim:** Validation, Data curation. **Ran Zheng:** Validation, Data curation. **Gil Ho Yoon:** Supervision, Funding acquisition, Conceptualization. **Bing Yi:** Writing – review & editing.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No data was used for the research described in the article.

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